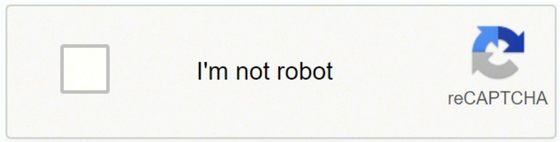

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72430875648 83552748508 17037593.851064 110661675993 1949010.7473684 16207939.824324 43201625.978723 41429051076 293770305 17468547.946237 23489506.04878

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A cryptarithm is a genre of mathematical puzzles in which the digits of an arithmetic problem are replaced by letters of the alphabet or other symbols. Next, you will be guided how to create a collaborative environment in the classroom to get the most out of each allotted period. Actions speak louder than words, so be the curious, connected, and collaborative learner you'd like your students to be. In game theory, this is the mathematical representation that one player's gain is equivalent to another player's loss, so in nonzero sum games, everyone improves. Then (24 − n − 2)n − 42 = (24 − n + 4)(n − 6), yielding n = 13 and m = 11. The unit is accessible to middle and high school students. When you start working with your students to create an environment where questions are a key component, then their curiosity will be unleashed, and they can begin to master the topics to which they put their energy, play, and thinking. These exams are multiple choice, and high scorers at the AMC 10 and AMC 12 level are invited to compete in the AIME. The best way to complete the problem set is to make sure multiple students have a voice. This distance is considered only horizontally and vertically. Motivated by this question, Fermat asked in 1643 for the case of three given points: Which point would minimize the sum of distances? Titu Andreescu and Branislav Kisacanin, Math Leads for Mathletes, Book 1 (Providence, RI: American Mathematical Society, 2014). Thus, one of (p, q, r) is a multiple of 3, and because it's prime, it must equal 3. Flocks of birds often fly in this triangular formation. It's easiest to create the diagram working backwards with the student who has all three pets. There needs to be a fundamental shift in approach and exposure to a range of problems that are harder and more interesting so that students can see where math can take them. Further, it's not just about students seeing their own strengths – problem solving and competition math train students to notice. If one looks closer, this is actually a Lo Shu magic square with 19 added to each number in the square. Positive peer pressure (yes, there is such a thing, even though we only hear about the negative peer pressure). The third polygonal number counts all vertices and points in the resulting figure: Triangular Numbers Square Numbers Pentagonal Numbers A triangular number is a polygonal number: a number that can be represented by a regular geometric arrangement of equally spaced points. Find the greatest possible value of n. To have learning and growth, the gains need to be significant and there should be pain relievers for the potential negatives. Feedback loops are essential so that teachers know what works and when to pivot. The sum of the areas of the n triangles whose bases are the sides of the polygon is equal to the area of the polygon. What are your favorite problems, and why? Math competitions can take this approach even further by: Uniting students toward a common purpose, which is succeeding in competitions and improving their scores. In other words, when developing a roadmap for the school year, you want a few main goals, but the strategic initiatives to get there must be flexible enough to change with the altering landscape of your class and how it changes over time. The more practice students have explaining their work in a supportive environment, the more they can carry that skill into the future with the confidence that they know how to succeed. Other times, if the initial question asks them to prove that statement A implies statement D, in my head I break the problem into a chain of implications, something like A implies B implies C implies D. However, when computing became abundant and the constraint became humans, this meant that talent and agile responses were more important to productivity, and, subsequently, processes needed to be maximized around human resources and not the machine. What can you control about the fear? Take note, this will go beyond the standard school curriculum, e.g., what you would need to pass an algebra class, and instead will show the additional skills you need to implement a problem-based algebra unit. Be vulnerable and make mistakes in front of your students, asking them for help to solve the problem, which demonstrates it is about the process and not knowing everything from the start. List the first six terms of the sequence. Solution The only addends are 1 and 1; made into reciprocals, 1/1 + 1/1 ≠ 1. Assessment Problems 1. I try to ensure the kids are learning by getting quick feedback in class. Solution Since T5 = (15) and S5 = (25), 2T5 − S5 = 2(15) − (25) = 5. Just as a consultant may not know a business as well as the company, consultants are still hired because they offer a fresh perspective and have the experience and knowledge to ask the right questions so the company can grow. I think problem-solving ability and logic are the reasons to teach mathematics, besides the pure beauty of mathematics – all are tough sells to students, but that's what I try to sell them on, because no, they may never use complex numbers after they leave my class, but they will have to learn how to operate within a construct and use logical reasoning to make decisions and solve problems. As defined by IDEO founder David Kelley, design thinking is “a human-centered approach to innovation that draws from the designer's toolkit to integrate the needs of people, the possibilities of technology, and the requirements for business success.” Why are these systems so popular now? Thus, the last digit of the sum 5! + 6! + • • • + 100! is zero. Summary Using a protractor, draw some similar triangles and test to see if the angles always add up to 180°. Thinking about problems is very time consuming, and I try my best to give them the problems for which the process of discovering the solution is the most enlightening. Prove that there are three points located in a 1 × 1 square. The real problem for me, though, was to find the mistake in my first idea (based not only on intuition but on seemingly wellknown mathematical facts). The only way this can happen is if A = 4, and a 1 is carried over from the previous column during addition. This logic problem is exactly what an adventure should be: interesting, engaging, and having depth and layers. Alternatively, let y be the eleventh integer. Don’t get discouraged if you have trouble at first. The next term in the sequence is 127. In particular, one of them is 3, so X must be 2 or 4. Many times, students are nervous to provide quality feedback, especially if their name is attached to it. List the first five terms of the sequence. If a1 = n, a2 = n, ..., an = and then n2 is a nice number. The ratio of two corresponding segments is called the coefficient of similarity of the triangle. When a teacher is the one solving a problem on the board, it can seem so easy to do, but when the students must solve a similar problem independently, they can get stuck. Toothpicks Math OVERVIEW For many years, scientists tried to measure the benefits of puzzles on the human mind. Math Kangaroo. Regístrate Gratis Accede a nuestras Descargas Privadas From the Training of the USA IMO Team.This book contains 104 of the best problems used in the training and testing of the U.S. International Mathematical Olympiad (IMO) team. Moreover, one can prove the following: If then k = 1. Here are the top five characteristics of a problem-based learning curriculum as detailed in this book: 1. Do you have a strategy for solving the problem? Students cannot be over the grade level or age level for the exams; however, younger students are eligible to take them if their math skills are up to the challenge. See Collaboration; Community; Competition Time, 239–243 Timing, 55–56, 60, 78 Toothpicks math, 163–165 Top-down approach, 36–37 Torricelli, Evangelista, 145 Trade, 113 Training, 15–16 Transformation, 239 Trapezoids, 90–91 Trial-and-error, 163 Triangles: angles in, 180–183; area of, 235–236; combinations with, 248–249; congruent, 168; definitions of, 177; equilateral, 241; Heron's formula for, 219; midlines of, 90; Pascal's triangle, 86, 245–253; prime numbers and, 201; similarity in, 90–91; triangular numbers, 199–204, 206–210, 246; triangulation, 239, 241. The figure has an area of 45. There is both an individual and team category. They aren't as fast to memorize, but once they have the idea firmly placed in their fabric, they remember it longer even though it may take longer to learn. If triangles ACD and CBA are similar, find the angle measure of ∠EAC. Each problem requires proof articulation, and high scorers can qualify for AIME (see below). The sum of four consecutive integers is 46. The smallest x such that 104 < 11x is 10. Find ∠D. Example 5 Using Pick's theorem, calculate the area of the polygon in the following figure. So (n + 1)-gon after triangulation will leave us n − 2 + 1 = n − 1 triangles. When you know each participant's pains you can: Pivot within the system. The book further stimulates students’ interest for the future study of mathematics.In the United States of America, the selection process leading to participation in the International Mathematical Olympiad (IMO) consists of a series of national contests called the American Mathematics Contest 10 (AMC 10), the American Mathematics Contest 12 (AMC 12), the American Invitational Mathematics Examination (AIME), and the United States of America Mathematical Olympiad (USAMO). Once a guess is made, one can verify it with the given terms. It's when a ceiling is placed on exploring talent that a student can become bored or disengaged. Proof We use the acquired knowledge from Viviani's first theorem. Tetrahedral formula: (n(n + 1)(n + 2))/6. Selection to the USAJMO is based on the USAJMO index (AMC 10 Score + 10 × AIME Score). Meaningful problem solving takes practice. Proof Connect point P to the vertices of polygon. The perimeter PADCBP = 6 + 2x = 8.5. 6. 8.5 9. Hence n × 135° = (n − 2) × 180° or n × 45° = 360°, yielding n = 8. The answer is 60. DEFINITIONS Area. The sum of the internal angles of an n-gon is (n − 2) × 180° = n × 180° − 360°. Purple Comet! Math Meet contest, Middle School, 2018, problem 4, . Determine which box requires more wrapping paper to wrap the chocolate. Make good use of transition times at the start or end of classes to capture attention or provide a fun mental break with a collaborative challenge. The amount of a surface or plane that a figure covers, expressed in units. In Chapter 4, we discussed how to create a classroom environment where questions are welcome in the segment “Mastery through Inquiry.” Many of these same techniques, slightly modified for the parent, can be shared so that parents can guide their kids to asking good questions about the problems they may be stuck on. It's not about winning; it's about individual improvement. So many times in life, we go through the motions and follow the steps without really questioning. Our brains can only hold on to so much new information at a time, and with the pacing requirements of various districts and schools, taking some time once a week to Review ensures that the students don't just “pump and dump” information – meaning they take in new concepts for short-term proficiency and then quickly dump the information for the next new lesson. The next consecutive number is x + 1, x + 2, x + 3, etc. On the other hand, from the top rectangle A'B'C'D′, by the same theorem, f 2 = b2 + c2. Student B. Again, start small, but try and identify one or two colleagues with whom you can meet with on a regular basis to share ideas and explore different concepts. Math competitions tap into the human need for connection and purpose, but only if the environment focuses on the process and not the outcomes. O, T, T, F, F, S, S, ... 10. Further, magic squares are a good introduction to mathematical constructions, providing an accessible example utilizing arithmetic to create these constructions. Create flash cards that have one role written on each card, e.g., leader, innovator, thinker, and idea facilitator, and have students randomly draw a flash card from the pile. Write 450 as the sum of: a. CHAPTER 12 Triangular Numbers Learning Objectives Students will be able to use triangular numbers to solve problems. The composers, in order of birth year, were Bach (born in the year 1685), Mozart (born in the year 1756), and Beethoven (born in the year 1770). The child who is making up worlds, whether they write stories or design games or play with languages, is a creative kid. High standards. Recognize That Everyone Is Good at Math In this book, so much emphasis is placed on process over outcomes – and that is even more true with mathematics education as a whole. 8 3. Further, it takes time to implement a problem learning curriculum, but the time required can be minimized if the teacher is a part of a professional learning community where ideas can be shared and strategies discussed. Have students roleplay asking questions. I have been blessed to have been in the right place at the right time. It was created by H. Although I was constantly trying not to rush and most students could follow my explanations, I was showing too many examples and not leaving enough time for them to think. Some people can immediately visualize the solution, and then they work backwards to see how they did it. MATHEMATICAL GAMES For stronger bonds and cohesion with PBL, making time to play is very important, but it doesn't have to be frivolous. One way to solve this problem is to count all possible teams when A participates (and therefore B also participates) and separately if A does not participate: Another way to solve this is to count the number of teams in which A participates but B does not and to subtract that number from the total number of teams: How Have Your Teaching Methods Evolved Over Time, and Why? Properties of the midpoints of the sides of a quadrilateral: The midpoints of the sides of a quadrilateral are vertices of a parallelogram. Participation in the AIME and the USAMO is by invitation only, based on performance in the preceding exams of the sequence. Teaching is an opportunity to inspire and guide, but that means diverging from the conformity required in today's education system and allowing students to take intellectual risks and, yes, fail. Even though the initial terms of a sequence do not determine that entire sequence (there are infinitely many sequences starting with an initial set of values), knowing the first few terms may help one make an educated guess. The next number is 4. We draw a parallel line through B and let D be the point as shown in the figure above. Students find their role in the group and can add their ideas to the mix so that synergies happen. Are they driven by decisiveness or purpose? Why is a collaborative problem-based approach worthwhile? Steiner Minimal Trees on Sets of Fourt Points, Discrete and Computational Geometry, 2e, 401–414. Find the area of the region between the two circles. Factorial. And here is how I present my “solution” to the students. This unit is accessible to middle and high school students. v=dAyDi1aa40E]). There are a myriad of activities and subjects constantly vying for a student's attention, so why should they devote time and energy to math competitions? For example, MATHCOUNTS has school, chapter, state, and national level competitions where students can meet other mathletes who enjoy solving interesting problems and find true peers at their ability level. If students can be exposed to the tough problems in mathematics early on, this rigorous training will make other challenges easier to face. At the AwesomeMath Summer Program, instructors from across the globe fly to the United States to work with students to improve their competition math scores utilizing a PBL approach. What mission can these students easily unite behind? Find the value of θ. Two angles that added together produce an angle of 180°. If GE is perpendicular to AB and ∠ADC = 70°, find the measure ∠GCD. Rotate leadership. This means, you must also be open to the mission evolving and changing as the needs of the stakeholders evolve and change. The midline of a trapezoid: The midline of a trapezoid is the segment connecting the midpoints of its legs. There are shiny treasures to repost, ‘today-I-learned’ surprises to ponder, wise checklists to save, heartfelt polemics to debate—and so many kind math friends to meet!” Dr. Maria Droujkova, Founding Director of Natural Math “I believe the most important goal of education is acquiring the ability to learn on your own. So, how do we represent these numbers algebraically? We have a + b + c + d + e + f + g + 2(α + β) = 900°. Jossey-Bass books and products are available through most bookstores. We mark six points in A, the vertices and the midpoints. CE 120) was a Greek biographer and essayist; he was a contemporary of Nicomachus of Gerasa, an important ancient mathematician best known for his works Introduction to Arithmetic and Manual of Harmonics. Luckily, there are some common factors that play a role in the success of every student, regardless of where their life journey takes them, e.g., mathematician, dancer, engineer, artist, biologist, linguist, computer science, or any combination of areas! These four quadrants apply when pursuing problem solving and other areas of life as well. Knowing that they are not alone in their love of knowledge and curiosity. Then, (k + 1)a + (k(k + 1))/2 = 100. Figure 7.2 Bolzano's theorem says that if a continous function is sometimes positive and sometimes negative, it must be zero as some point. First, you need to have them stop and reflect about what puts them into flow. It's been used to pump and dump, namely, shove as much information as possible into a student so that they, in turn, can dump that information on to a bubble sheet of multiple-choice answers. Praise. What is the sum of last two digits of (1! + 2! + … + 2019!)? The following relation holds: |A∪B| = |A| + |B| − |A∩B| The argument for the proof of this relation is very simple: When we count the elements of A∪B, we count two times the elements of A∩B; hence, we have to exclude once these elements. Randomize teams of students to work together so they can build relationships. It thus follows that X = 1, Y = 8, and Z = 1. Consuming with purpose (active) is a lot different than passive consumption. Not reading carefully (missing information) 3. Let a = n2, then b = (n + 1)2, c = (n + 2)2, d = (n + 3)2. However, at the end of the booklet that was provided, I found more interesting and complicated problems, and I was excited to try and solve them. Or that zero started as a placeholder and when it changed to be a number, it elicited great fear – how can you give substance to nothing? Students can also consider the perimeter of the shapes as the interior points increase or decrease. The note-taking column is wider because this is not only for lecture notes, but also a space to relate, reflect, and revise. Students aren't learning how to think, work together, or find challenging opportunities. The groups don't compete against each other, but at the conclusion of their work, the whole class should be brought together for a thorough discussion. Then, because the last two displays use the same number of pieces, we have In other words, we have m(n + 2) − 82 = (m − 2)n − 42, yielding m = 24 − n. You're Reading a Free Preview Page 76 is not shown in this preview. Students have the chance to solve and show their reasoning for the problems, providing an opportunity for young problem solvers to try their proof writing skills. PRESENTING PROBLEMS ON THE BOARD Much of PBL revolves around students as teachers, meaning that they need to be presenting problems on the board as a regular part of every class. My favorite problems are the problems that in the end require two or three lines to solve, but you first need to spend 10–15 minutes thinking about where to even start. 88, 89, 90, 91, 92. A dyslexic student may have difficulty focusing on word problems but excel in three-dimensional thinking, geometry, and visual proofs. That is why problem-solving-based curriculums manage both the short and long view of mathematics education – one foot must be firmly planted in learning in the here and now while the other steps forward toward the future and how to apply the knowledge gained. They built landing strips, airplane models, and headphones, all from bamboo and wood, to mimic the motions of what they observed from the advanced culture. Creating something new is increasingly difficult and, again, that is okay! This section is all about rewiring your brain to think differently through the creative process so that your teaching and problem solving will improve. Students are exposed to discrete mathematics, crossdiscipline subjects, proof-writing skills, and much more. Inductive reasoning problems start with observations of patterns or trends and then generalize that data to arrive at a conclusion or conjecture: Last night there was a party, and the host's doorbell rang 20 times. States that the sum of the distances from any interior point to the sides of an equilateral triangle equals the length of the triangle's altitude. Do there exist five distinct prime numbers for which the sum of any three of them is a prime number? Two triangles are congruent; for example, if their sides are of the same length and their internal angles are of the same measure. The rectangle ACC'A′ is pictured here: By the Pythagorean theorem, g2 = a2 + f2. From all of them, triangles with triangles inside, the greatest number of triangles is 7. Do you understand the problem; what information is given, and what is being asked? The lecture model is a faster way to disseminate information to a large group of students, even if it isn't the most effective for long-term retention or increasing test scores. 22 = 2 × 10 + 2, and we showed 10 is nice; therefore, 22 is also nice. When we divide 721 by 7 we get 103. Whenever learning becomes stale, rote, check the box, or – gasp – timed, then brains turn off and students become trained to just go through the motions. As mentioned in Section 1, good problems should have the following characteristics: They take several steps to solve. Study the truth of this statement for all positive integers less than or equal to 7. But the magic square in this example looks different. Pentagon: 180° − (360°/n) = 180° − (360°/5) = 180° − 72° = 108° × 5 = 540°. As a facilitator, instead of answering these questions, you will instead question them. So I call these things cargo cult science, because they follow all the apparent precepts and forms of scientific investigation, but they're missing something essential, because the planes don't land. You're Reading a Free Preview Pages 53 to 57 are not shown in this preview. American Invitational Mathematics Exam (AIME). That's not to say that when you reach adulthood, KAPOW! you now have all the answers to life. Congruent. A rectangular cuboid with integer side dimensions such that the face diagonals are integers. Deeper When made to think critically, they learn concepts at a learning deeper level than they would in a broadcast environment. Families are complicated and come in many different forms: Various marital statuses such as divorced dual custody, singleparent households, married, remarried parents with other children, etc. What Is Your Personal Approach to Teaching Problem Solving? Archive for History of Exact Sciences 68 (3): 327–354. 8, 10, 12, 14. You can also observe for yourself how a student handles a new challenge in class. d. I spent four years there with the last three years under the guidance of college faculty members as my mathematics teachers. You don't You can practice know if presenting problems you'll get to smaller groups. RELATE, REFLECT, REVISE Teachers and students need to go through this exercise: How does the math relate to other areas, reflect and gather feedback on how well the lesson worked, revise and iterate to improve, and then share what you've learned with your class and/or community. Just like when we asked the question of students, “How much time do you spend consuming things other people have made, and how much time do you spend creating something original?,” we could ask the same of ourselves. And for this, motivation and perspective/context are very important. The least common multiple of my number and 9 is 45. You can have your students talk out loud about how they are thinking about the puzzle, or to tell you what steps they took if they managed to solve the puzzle quickly. Mistakes make us better and help to develop a growth mindset when viewed in the proper light. Solution Consider vertex An and let us triangulate the polygon into the following triangles: AnA1A2, AnA2A3, AnA3A4,..., AnAn−2An−1. Find the last digit of (1! + 2! + ... + 100!). Furthermore, having students present improves their confidence and presentation skills, which are so critical in the modern world. When a class is only 45 minutes long, how can you introduce a new concept, e.g., divisibility rules, and still have meaningful collaboration amongst students for how to solve problems related to the concept? The diagonals of a rhombus are perpendicular. Core Values Having core values allows the instructor to create an environment conducive to learning and provide a guide for students so they can self-check their behavior and attitude in a way that is not personal. CHAPTER 1 Rewards for Problem-Based Approach: Range, Rigor, and Resilience Range Ignites Curiosity As educators, we understand the importance of depth and breadth in learning. You're Reading a Free Preview Pages 17 to 24 are not shown in this preview. Gudder Mathematical Relevance Why should math be relevant? How can parents be provided a map of engagement if they don't have a background in PBL? Certainly, math isn't the only area where constraints can be creative. What is important for me, I never think about “training” for the Olympiad. You test strategies, take the best practices learned from those approaches, and iterate to the next level while keeping the objective for the lesson in mind. And, maybe most importantly, we learned because we talked about the concepts, we worked with others, and we learned because we teach! Collaborative work helps emulate this environment and helps students that know a bit more really secure concepts by trying to teach others. Does the solution tie back to the lesson? Let ABCD be a quadrilateral such that ∠A = 70°, ∠B = 80°, and ∠C = 90°. My luck was that I never considered that “mathematics.” I understood way before I knew how to explain it that mastering computations is necessary for doing mathematics as mastering a language is necessary for sharing ideas with other people. Listening is just as critical as providing information so that ideas can build on each other. But instead of digging into the literature, I was eager to find where the mistake (the catch) in my reasoning was – in other words, where the dog lay buried (as the Bulgarians would say in this case). But we also know 4 = 2 × 1 + 2. It took me a while to understand that there is a difference between students being able to follow me as I go step-by-step from first to last and them being able to produce a similar long sequence of steps in order to solve a slightly different problem. However, when you design to the ends of the bell curve in the class, you create an active learning environment that allows all students to excel. Then This goes to prove that both 2n + 8 and 2n + 9 are nice numbers. You can find other mathematics teachers in your area and start a circle for the kids. So, every test had a good portion of challenging questions. As adults, many of our big-life questions have been answered, the most important of which is a clear sense of purpose or self. By the time they managed to prove that, they will be already working to complete the chain by proving C implies D and therefore finishing the problem. We will now work on making it a reality. The sum of these three numbers would be divisible by 3, and therefore not a prime number. In the rightmost column, two H's add up to D. Schools register in the fall and there is a perstudent registration fee for students going to the Chapter Competition. It is scalable so that problems are in a range to reach all levels of students and promote their individualized growth. These competitions are of great interest in Romania, and what I found unique to other countries is their scale. Factorials grow rapidly. We know that n, then . Finally, the Summary section at the bottom of the page is a space where students can write one to three sentences on what they believe is the main goal of the lesson. Thus, the greatest integer for which n! ends in exactly in 33 zeros is n = 139. To accomplish this goal, you can follow the same methodology used by doctors to diagnose wellness. While this is a simple exercise rather than a complicated problem, it illustrates a larger problem in mathematical thinking. As he points out, there is no such thing as an average student just like there is no such thing as an average genome, average memory, or average cancer.6 And yet, our entire education system is built around the average student (e.g., these are the things the average 8th grader should know). Then the area of the polygon can be found using Pick's formula: K = I + b/2 − 1, where I is the number of lattice points inside the polygon and b is the number of lattice points on the boundary of the polygon. 11. Some days are still skills days, but I try to really point out what the end game of the skills are. Journal of Combinatorial Theory. Therefore, the probability is Solution 2: Recall that Meow pressed different keys, so it is randomly picking a four-element subset from a 26-element set. Solution They are not; 5 is not a nice number because when we add its reciprocal addends, it doesn't equal 1. You're Reading a Free Preview Pages 8 to 12 are not shown in this preview. The sum of the first n triangular numbers is the nth tetrahedral number. Find n.6 Solution The sum of all entries in the magic square (including the number 37) is so the sum of the entries in each row is Then implying It follows that n = 16. When they have something fun and quick to work on from the very start as they enter the room, transitions will be smoother. What do they think the teacher wants them to take away from the lesson, and why? This program is for students 13–18 years old where they are given the opportunity to become entrepreneurs and “apply innovation, science and technology to solve problems with global impact.” The challenge values and develops skills in collaboration, creativity, critical thinking, and communication. Therefore, n = 7. The product of two consecutive numbers is 12. 5.5 8. By definition, then, an integer n will be called nice if it can be written as n = a1 + a2 + … + ak, where a1, a2, ..., ak are positive (not necessarily distinct) such that 1/a1 + 1/a2 + ... + 1/ak = 1. They can be represented with counting or algebra, and many clever relations and identities can be found. A common thread when people are asked this question is that the teacher believed in them, so they were challenged to be better and felt capable of reaching higher expectations. Currently, math education in middle and high schools is a series of exercises with easily obtained answers, e.g., find the perimeter of a square, training students to do what a computer can do better. It takes the shift away from outcome-based learning (grades/test scores, rank, grade point average [GPA]), which is a fixed-mindset approach, to learning for mastery, where students challenge themselves to improve every day (growth mindset). A triangle can be transformed into a rectangle. When the work is about the process and not the outcomes, then you can focus on making the process the best it can be, and that means knowing how to ask the right questions. From the similarity of triangles, we have m/h = h/n, which implies that h2 = mn. Students too often carry the belief that success in math is based in rote memorization and drilling problems. As a matter of fact, because of the class work solving interesting problems, we didn't maintain the math circle in the high school years. For example, the teacher will offer a brief discussion of a topic and then the students put their brains together to come up with solutions. The discussion of this theorem incorporates many techniques from geometric proofs: we draw extra lines and structures to support our argument and draw conclusions from geometric observations. You aren't dictating behavior, you're encouraging them to think more long term about the networks they hope to create in the future and how those networks are started and nurtured. I then spend a lot of time thinking how to present the theory and examples so that the context of the topic is explained very clearly and all the solutions to the problems we cover feel natural and not like an “out-of-nowhere” trick. Note that Note that the Pascal triangle has axial symmetry; then, That is, We know that sum of entries of nth row of Pascal triangle is 2 n . Encourage students to draw figures and pictures when they record information as it will tap different areas of the brain to create a fuller picture of what is being learned. There is a per-student registration fee of $21 and the school would need to become a test center for the program. There are plenty of problems that you may come across in your own work where you can share the process of how you solved them with your child. Moreover, the pigeonhole principle provides us with a clear picture of what is going on. Encourage all questions. I've found that I most enjoy working with small groups and allowing students to dictate the conversation as they work through a concept. Two courses stood out: math analysis (a beginners' version) and Galois theory. Numerous problems involve consecutive numbers, such as 1, 2, 3 or 9, 10, 11, 12. The purpose can be defined by the problem at hand, a strategy that should be employed, and/or a concept that must be applied. 4 5. As a result, the students are presented with a refined overview of the theory and also with a large subset of the key strategies for that topic, both of which are typically acquired with years of experience. Why are there so many math enrichment centers popping up across the country? Chen, X., and Du, D.-Z. Try new things and give yourself grace when you need to pivot and change your plan. In music, they may enjoy composition more than perfection with playing a piece. This mental exercise of comparing or judging is not a bad one, but it is incomplete without understanding. The construct of the Hero's Journey (a.k.a. Monomyth) applies well to those studying problem-solving and mathematics. Since 102 + 242 = 100 + 576 = 767 = 262, we find that the perimeter of this rhombus is 4 × 26 = 104. Game theory. Problem solving is about working around obstacles to understand the unknown. Know your students. Skill gaps If students have gaps in the foundational material necessary for the class, they can feel behind and, in turn, disengage. The given sequence is an arithmetic progression with the initial term a = 1 and the common difference d = 2. A student who was the why kid, “Why is the sky blue?,” “Why do people get sad?,” “How do bumblebees fly?” may be the type of child who wants to know the story of mathematical concepts such as those attributed to Pythagoras. There is a lot to absorb when embarking on a problembased approach, and this section intends to provide you with the necessary resources, so the information is in one handy location. (Hint: Start with an equiangular triangle of side 9 and cut corners!) 5. Most simply, the sum of two consecutive numbers is a square number. And lastly, to my mother-in-law, Sandy, and my sister, Kelly, for being early readers and emotional support. Take your time! Very few contestants can solve all the given problems. Pythagorus's story is full of interest and intrigue, For example, he was convinced the whole universe was based on numbers, and as such, the planets and stars must move based on mathematical equations. When teachers tap into a professional learning community (PLC) such as the MTC, they will be able to trade ideas and been there, done that experiences to streamline and improve their classroom management as well as their lesson plans. Math games, problem hunts, and mathematical modeling lessons are all examples of ways to add physicality to the classic design of the math class. Nothing would make her light up more than a new perspective when thinking about a topic, and she could immerse herself in the tiniest of details. Prove we can choose four students who were born the same day of the week. CHAPTER 10 Consecutive Numbers Learning Objectives The students will explore different consecutive forms of numbers. This change has happened mainly due to pedagogical research that shows how active learning is so much better for students, and especially for students of color. This doesn't, however, mean there aren't pains to providing the at-home support necessary to embrace the problem-based approach. It helps fight imposter syndrome among historically underrepresented groups and seeks to create a place for students to feel safe and to enjoy playing with mathematics. That said, retention, understanding, and learning are deeper, but some parents may feel uncomfortable with the pace. So how do you help students to know how they think so that they know what value they add with collaborative problem solving? We seek out recommendations from others, but in the end, the autonomy to choose and research on your own trip makes the vacation more customized and more interesting. Even mathematicians and researchers haven't come up with a definitive answer, but in this book, we believe problem solving has the following characteristics: Problems take several steps to solve. So we need to find how many tens are in the product of 25!. Having the ability to inspire your students to be better problem solvers will affect their lives for years to come and, in turn, they can aspire to rise to the example you have set for them. So let the holes be five pairs of consecutive numbers: (1,2), (3,4), (5,6), (7,8), (9,10). You're Reading a Free Preview Pages 17 to 19 are not shown in this preview. That is not to say that they are incapable of deep thought, but rather, asks how mathematics can compete with all the other distractions that life throws their way. If the strategy doesn't work, then go back to step 3 and amend your approach. Solution No matter what you do, you have two factors (n(n + 1))/2. Series A 32: 396–400. Now, she can chose 11 numbers such as 720–b × 41b, b = 0,1,...,10. When you notice a loss of focus, taking just one minute to breathe rhythmically with the class can offer a much needed reset. This principle found its own niche in the vast world of mathematics. Mathematical Reflections is a free online journal aimed at high school students, undergraduates, and everyone interested in mathematics. As they grow in knowledge, they can become the mentors and helpers for others on the path. Acknowledgments Special thanks to Navid Safaei and Alessandro Ventullo for their time in reviewing the mathematical content for this book. Then, they will cross the threshold, have the confidence to solve problems, and be more willing to take intellectual risks. This approach also breaks kids free from the bonds of staying still in one place. Almost all researchers agree that puzzles help cognitive processes. (Check out the full units on Triangular Numbers and the Pythagorean Theorem Revisited to learn more!) However, just like Pythagorus felt that the number 3 was an ideal number because it has a beginning, middle, and end, therefore representing harmony, it's best if the students reflect and come up with at least three core values to ascribe to themselves so that their true complexity of thought and uniqueness can shine through. When running a PBL classroom, you can also find peers and colleagues in other disciplines who follow a similar teaching method such as teachers of Computer Science, Physics, Statistics, etc. A fun problem that works well in a group is the following: The diagram shows a polygon made by removing six 2 × 2 squares from the sides of an 8 × 12 rectangle. You're Reading a Free Preview Pages 55 to 63 are not shown in this preview. Practice receiving. Don't confuse talent with being good at math. Olympiad questions can seem impenetrable to the novice, yet most can be solved with elementary high school mathematics techniques, when cleverly applied.Here is some advice for students who attempt the problems that follow. Figure 7.2 Bolzano's theorem says that if a continous function is sometimes ... The answer to guiding a child to make good choices is by, again, giving them control (autonomy) over their outcomes. Definitions Polygons. It gives parents a map of engagement for working with their child and their teacher, meaning that parents can help the student develop what questions they should ask the next day to increase mathematical fluency. This is a fun problem that is collaborative and sparks curiosity! The figure was made by gluing together five nonoverlapping congruent squares. Know your approach. Add in lack of sleep due to juggling so many commitments and it is even more difficult to be connected and engaged. I vary it up. 19. Example 2 If find the value of n. There are books in which trapezoids are allowed to have one pair of opposite sides parallel. So much of math education today is about waiting: Wait until high school, and then what you've been learning in middle school will be useful. The Math Rocks curriculum, developed by Dr. Andreescu in 2008–2010, is still going strong in the Plano, Texas, school district for elementary and middle school students. Du and P.M. Pardolos). Analysis of, for example, classroom management can help to free up time that is wasted in class to get the most out of each moment and effectively learn the lesson in conjunction with the collaborative problem-solving component. Determine which box contains more chocolate by comparing the volume of chocolate in each box. Now, it is clear that he has 31 students in his class. Cryptarithmetic OVERVIEW Cryptarithmetic is the art and science of creating and solving cryptarithms. The last digit of the sum 1! + 2! + 3! + 4! is the last digit of 1 + 2 + 6 + 24, which is 3. If they are connected with the roadmap, they will have more of a vested interest in meeting goals. Throughout this chapter you will be exposed to one of his secrets: You can be a mathemagician, too! Do unto one side as you would do unto the other. In many of my classes, I assign a project with the main goal of having students use the concepts learned in class to solve a real-life problem. Why not begin this process in mathematics courses? All sides are the same length. Start playing with ideas, manipulatives, history, etc., so that students open up and are more willing to question and extend ideas. Based on discussions with teachers, parents, and students, these are a few gains and pains charts for a problem-based approach in the classroom. Students may view the following websites to review simple sums and triangular numbers. These roles will take on the characteristics of the individual, and therefore you are only giving them the name of the role and then letting each student define it themselves so they can succeed. What makes the difference between perseverance and giving up? Why or why not? Prove that the triangle A could completely be covered by 4 of them. Examples: 342 = 30 × 38 + 42 = 1156 482 = 46 × 50 + 22 = 2304 752 = 70 × 80 + 52 = 5625 Solve, using the above method: 572 632 892 1132 Solutions 572 = 54 × 60 + 32 = 3249 632 = 60 × 66 + 32 = 3969 892 = 78 × 100 + 112 = 7921 1132 = 100 × 126 + 132 = 12 769 2. The derived formula does not require math proficiency beyond elementary school and can be easily verified with the help of a geo board. You may have individuals who want to buy the package vacation versus those who want to wing it and see where the vacation takes them. Rotate. For many students, this is a very scary adventure threshold to cross, and as their guide, you need to know how best to inspire your students to take this step. Purpose with Competitions Problem solving provides purpose through meaningful and relevant challenges. And when students share insights, they are of course sharing other important things as well, such as curiosity, questions, and their enthusiasm for problem solving. Solution We dissect the square into 100 unit squares. 3, 3, 5, 4, 4, 3, 5, 5, ... 13. This is a generation that is used to every aspect of their lives being recorded and shared, and that can lead them to a false sense of security. Pat applied Pick's formula to find the area of a polygon but mistakenly interchanged the values of I and b. I make no question but you will readily allow this square of 16 to be the most magically magical of any magic square ever made by any magician. Since I is at least 1, we have SI − FO ≥ 2. See USA Junior Mathematics Olympiad USAMO. If they still need the manipulatives (pictures of the animals), they can cut them out of paper and keep track. Technique Model the behavior. Students need to present what they've learned to the class so that they can all grow together and gain the confidence for bigger challenges. WHAT ARE GOOD PROBLEMS? Let A = {x|x is positive integer, 104 < x < 1300, and 11|x}. Create peer-tutoring alliances so stronger students can help classmates who need more foundational help. Big Think had a wonderful article1 on dealing with anxiety in organizations that applies very well to math phobia in education. Helping students bridge the class with their future jobs, or their lives in general. Algebraic proof of Problem 9: 10. That integrity can take many forms, such as respect for the process and the players, honesty in all actions, and the quest for knowledge. Therefore, c ≤ 2. They are overscheduled and have less freedom to play and explore. MAP OF ENGAGEMENT AND PARENT INVOLVEMENT An often-overlooked and yet robust learning community is the parents of your students. When students present problems on the board, the entire class along with the teacher can support their progress and help them with deeper understanding of the material. LEARNING OBJECTIVES Students will be able to find the area of simple polygons using Pick's theorem, which is a good introductory geometry topic that considers polygons with vertices that have integer coordinates. Daniel Pink, who wrote the book Drive, states that there are three things that tap internal motivation: 1. Prove that P6 + T6 − 2S6 = 0. Figure 7.11 One of two possible solutions with symmetry. The common thread is playful problem solving. The teacher would often ask if we wanted assistance, and we would usually answer that we wanted to solve it on our own. The only area where they are still exposed to a broadcast approach is education, and students' talent and abilities are subjugated to the machine of GPA, standardized testing, and rubrics that fulfill the goal of accommodating an idealized, nonexistent, average student. And finally, the math is meaningful, with real-life problems giving them connection and purpose. Then, they often make a conjecture, test their conjecture, and then informally or formally justify their conclusions. TIMING (LUCK) Timing has always been a key element in my life. It's about gently guiding and not instantly pointing out when they are on the wrong path (judging). Can they explain the process to a peer? How can I apply these ideas further?” Go back to the original problem later, and see whether you can solve it in a different way. For the same reason businesses need a mission statement and core values: To let their stakeholders (the students) understand what the class is trying to accomplish To help the student choose their role in achieving these goals To create an environment conducive to learning and growth toward the mission Mission Statement Mission statements set top-level goals for the class, provide a cohesive team environment, and allow students to assess their contribution to the group. The following Venn diagram shows how many times each group gets counted when we add the total members of all three sets:. In fact, much more is true: Any two polygonal surfaces with the same area can be transformed into one another by cutting the first into finitely many pieces and then assembling these pieces into the second polygonal surface. Roman numerals in modern day numbers IVXLCDM M = 1000 D = 500 C = 100 L = 50 X = 10 V=5 I=1 How to represent numbers that are less with 1 unit than V and X: 4 = 5 − 1 = IV 9 = 10 − 1 = IX How to represent numbers that are less with 10 units than L and C: 40 = 50 − 10 = XL 90 = 100 − 10 = XC How to represent numbers that are less with 100 units than D and M: 400 = 500 − 100 = CD 900 = 1000 − 100 = CM OBSERVATION In order to subtract one number from another in the Roman numeral system, you must place no more than one lesser value to the left of the greater number from which you are subtracting. The first ship returns to this port every 4 weeks, the second every 8 weeks, the third every 12 weeks, and the fourth every 16 weeks. In 2015, under the leadership of Dr. Loh, the US team won first place for the first time in 21 years! The last time the US team came in first place was under the leadership of Dr. Titu Andreescu in 1994. Always point out something positive about their response, even if it is to say, “This is my favorite wrong answer, because it is a great step towards understanding.” When students see they aren't being judged, and instead, being valued for all contributions, it will help create the open communication

the arithmetic mean of the first 20 triangular numbers. Studying dinosaurs? How or when did the student truly light up? When teachers guide students on the path of learning at their own pace in an inquirybased environment, they learn more deeply. Students work together to solve problems, not so they can be better than their classmates, rather, so all of them can improve and celebrate each other's progress. Important sequences (Left-aligned Pascal's triangle) The first column contains just one number while the second consists of the positive integers. With the five steps outlined below, you will be well on your way to launch PBL in your classroom, utilizing the provided set of mini-units (10–15-minute lessons) in Chapter 8 or full units (45–55minute lessons) in Section III. When letters are replaced by their digits, the resulting arithmetical operation must be correct. Definitions Triangular number. Dweck, Mindset: The New Psychology of Success (New York: Ballentine, 2016). If your child doesn't know where to begin, give them an example of a question you would ask. This may take longer, but the payoff will be so much greater. This gives the desired solution. Steps on how to build Pascal's triangle: At the top center of your paper write the number “1.” On the next row write two 1s, forming a triangle. Justify your answer. See also Approaches Learning objectives. (Note that an represents actual terms of the sequence (an)n≥1.) Example 1 Consider the sequence (an)n≥1, an = (1/n). This is a fun problem that a peer tutoring team can work on together. So, the question becomes “How do we inspire curiosity?” My approach to it has several facets: Tell students about famous mathematicians and scientists and their contributions. Test scores Knowing how to approach problems, deconstruct them, think critically, and imbibe in foundational material increases student test scores. It was first published in 1899, but only gained attention in 1969 through the popular Mathematical Snapshots by H. Public speaking is in the top three of biggest fears, so learning the coping mechanisms to have grace under pressure with a small classroom audience is essential. And of course, the Math Teachers Circle7 is a great place to find your PLC as well as your local math circle if there is one in your area. Create math stations where students physically move to learn new concepts, work in groups, or are reintroduced to manipulatives that solidify understanding. The Cornell method4 of note-taking utilizes this technique and is strongly recommended for the teacher and for the students. A classroom mission statement strives to accomplish the same goal and allows your students (the stakeholders) to understand the purpose of the class as well as where your heart is when teaching it. During class, I encourage a lot of interaction, regardless of whether we are discussing the theory or some problems, with one of the mottos being “there is no such thing as a silly question.” To boost the students' problem-solving skills, I challenge them to explain to me how they came up with the key idea in their solution. It was a huge flop because it didn't consider how the consumer actually used the products. Find the five consecutive integers with the same property (their sum is 90). In how many zeros does 25! end? We know that the nth triangular number is of the form (n(n + 1))/2. – Kathy We'd like to thank Amy Fandrei, our executive editor, for her kind guidance and for providing us with the opportunity to share our love of problem-based learning. They may have great focus but are slow with reading comprehension. Figure 7.8 Now consider how to minimize the sum AM + DM + MN + BN + CN,... It gives students a flavor of some introductory counting techniques. When you engage in the trade of ideas, everyone improves. Thus, the area is (14 × 12)/2 = 84. Two triangles that are the same size and shape. 9. Otherwise, by pigeonhole principle, two of them have the same value of a, and then one certainly divides another. When students spend an average of six to seven hours per day in school, why would they need additional math enrichment? The continuous Olympiad training soon became my main source of mathematical education. They aren't being graded; they are pursuing a worthy task, and that journey provides purpose as does the community in which the discoveries take place. Mathematical learning and reasoning are integral to the process of problem solving. CHECK FOR UNDERSTANDING Going back to the years presented on the board at the beginning of class, what are the answers? You can present problems to the class that are easier for you to solve so that you What can't you control? Let a = n, b = x, and c = x + 1; then from the Pythagorean theorem, we have This tells us that n2 must be an odd number, so n also must be odd. Following is a problem that requires a lot of thought and needs a wellconceived plan of attack, building resilience while also providing the thrill of solving! The diagram here shows a 12 by 20 rectangle, split into four stripes of equal widths, all surrounding an isosceles triangle. LEARNING OBJECTIVES Recall (or learn) the chess pieces' moves. Problem solving allows for a bottomsup approach where the goal is for each student to be better than they were the day before. We know that 17 = 4 + 4 + 6 + 3, and that 1/4 + 1/4 + 1/6 + 1/3 = 1/2 + 1/2 = 1. 18. In contrast, monthly contest problems may take best students hours or days of concentrated thought. The sum of the angles in a quadrilateral is 360°. Multiplying both sides by 10!, we find that Therefore, Now, we must calculate the sum of the elements of the 10th row in the Pascal triangle. Solution Assuming a + a + 1 + ... + a + k = 100. As the educator, you need to: Know yourself. How do you write 2019? … −2, −1, 0, 1, 2, … Even numbers. Prove that a + b + c + d − 5 is also a perfect square. It's about working together, not ranking, so that every member can grow and find their purpose in the group. After calculations we get 10! + 1 = 3 628 801. Find B′C′ and A′C′. Send and receive. Why is this important? See to download old contests. Prove that every other triangular number is a hexagonal number. A special machine was used to first graph the dots on the maps and then the theorem was applied. 5 2. This meant bridging that learning into other areas and seeking out the connections that led to understanding as well as contributed to the greater fabric of knowledge. Why Is a Collaborative Problem-Based Approach Worthwhile? Dr. Branislav Kisačanin: The social aspect of collaborative problem solving is immeasurably beneficial for developing young minds. Polygonal numbers. Working on puzzles? Teacher The teacher may not have the mathematics background inexperience necessary to teach problem solving at this level. It uses the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9. Roman Numeral Problems OVERVIEW The Roman numeral system played an important role in the era of mathematics. Solution It is important to note that 2020 is divisible by 4. Polygonal surfaces. The common thread is that they are all human-centric approaches. We can expand (2400 + 1)505 with the binomial theorem. As detailed in Section 1, it's critical to create a kind and supportive environment that respects the intellectual abilities of all participants along with interesting problems, like the one below, that are scalable for students of different mathematical skill levels. Students who score well on a qualifying exam, the F = ma competition, will move up to the next level. Assume to the contrary that all boys have different amounts of money. Find the perimeter of this polygon. Hence, 111 = 90 + x, that is, x = 21. It is held simultaneously at four locations: Penn State University, the University of Iowa, the University of Georgia, and the University of Nevada at Las Vegas. The school, district, and state may have pacing requirements, which dictate a demanding schedule of topics, making a broadcast approach more expedient even if it isn't more effective. Relating to geometric figures that have the same size and shape. A student-managed system where they are given different-colored sticky notes that represent different valued behaviors, such as one for listening or a good question, that they can put next to a fellow student's name on a chart is a great way to promote a positive classroom environment. That is, n2 − n − 30 = (n − 6)(n + 5) = 0. See MathWorks Math Modeling Challenge Magic squares, 159–163 Management, 93–96 Mandelbrot Competition, 105 Manipulatives, 17, 50, 163 Manual of Harmonics (Plutarchus), 201, 204 Map of engagement, 70–71, 120–121 Mastery: purpose and, 47; research on, 53; by students, xvii–xviii; vulnerability in, 49 Math Olympiad for Elementary and Middle School (MOEMS), 104 Math Olympiad Program (MOP), 117 Mathematics: AIME, 103–104, 106; AMC, 103–104, 105; chess and, 170–171; competition in, 103–106; discrete math, 2–3, 69–70; games with, 111, 114–116; math circles, 16–18, 112, 119; math clubs, 118–119; Math for Mathletes, 213; math kangaroo, 105; MATHCOUNTS teams, 97, 104, 122; Mathematical relevance, 14–18; Mathematical Snapshots, 165; mental mathematics, 155–156; New math, 68; phobia of, 37, 79. The largest possible value of TEACH + MATH is 99 999 + 9999 = 109 998, so we must have G = 1, I = 0. The real danger of not giving students adequate challenge and range to satiate their curiosity is that they will turn off on mathematics and learning altogether. Figure out where your child is struggling with the problem. Many will quickly respond, “1,600 pounds,” which is completely illogical if they have a sense of weight. Find five consecutive numbers that have a sum of zero. If 15 students play chess, 16 students play court tennis, and 10 participate in both these activities, then find the number of students in this class. If find the value of n . The paper needed to include: 1. Therefore, α = 80°. Regardless of the role a student chooses, all students grow their skills faster when collaborating toward a common goal than they would on their own, because when you engage in the trade of ideas, everyone improves. While the mission statement sets the course, a list of core values sets the behavioral expectations for the journey. Rectangular boxes and Euler bricks provide a case study in geometry and number theory. You may think this is a difficult problem, but all it takes is the ability to notice. This cross-pollination of thought and being able to communicate with others is how you create the fertile ground from where interesting problems can grow. Some schools are trying to combat the issue by creating an open period during the school day where students can talk about homework and work on projects, but this needs to be well managed to work effectively, and the best way to do that is to identify a few students to take on the role of group leader and cheerleader for the material. That's where a problem-solving curriculum can help. Through this quest for truth, you are training students to notice and self-reflect: Am I on the right path to solve this problem? They follow up with the patient to see if the problem has been corrected, and if not, they start again. Thus, d! > c!. When students see themselves as part of the team pursuing a worthy goal, then the relevance of the mathematics becomes the beacon that they choose to follow. The basic properties of the cardinal number are the following: 1. Provide an active learning environment. The word mistake conjures negative responses from the get-go and makes a student feel as if they are personally being judged, even if you are only evaluating their homework. −1, 2, 7, 14, 23, ... 7. How many integers from 1001 to 2000, inclusively, are divisible by 2 or by 3 or by 5? Example 2 Consider the sequence an = 1/(n(n + 1)). Do they ask for more clarification? An intersection point? This concept of average in education has been brought to light by L. Steinhaus.8 The theorem gives an elegant formula for the area of simple polygons. Use creative thinking to solve the questions asked. Students can't follow a program en masse and then all will be well. The first two terms are 1 + 505 × 2400, and every term after that is a multiple of 1000. Modeling this behavior is the best approach for guiding your students. “The buck stops here” is the border of leadership, and if a group is stuck, having a strong leader who can give direction with decisive action can really help. I also have more than a few exams (individual) which they take once in class and then take the same test as a take home. At what times did the student show real confidence and love for what they were doing? Each strip has a width that is ¼ the height of the entire triangle, so the small triangle in the top strip has area 120 × (¼)2 = 15/2. Currently, education is not equitably accessible to all students, with students from underserved populations and first-generation college students in particular facing additional obstacles to entering, navigating, and excelling in higher education. A problem-based learning curriculum provides all these points for making mathematics relevant. long and tangent to the smaller of two concentric circles. A benefit of a problem-based learning model is that it doesn't limit movement; many students process information by walking around the room or fidgeting, and it's important to let this happen as long as it doesn't distract others. We have: (n − 2)180° = (n − 2)α + 94° + 51°. Students can identify sequences and their impact in any area and write about it. When I began teaching, I heavily utilized a traditional lecture format. See USA Mathematical Talent Search Usefulness, in learning, 25 USNCO. Now, 15 years later, the shares are closer to 40%, 30%, and 30% and continuing to evolve in that direction. Therefore, parents may need to step in for support, but not all will feel comfortable doing so. In this way, we obtain |A∩B| = 57, |A∩C| = 44, and |B∩C| = 31. This means a 1 is carried over to the tens column. My first goal is that the student can read and understand the problems, then to teach the students how to be persistent and to learn to deal with difficulties and setbacks as well as when to ask for help and when to listen. At our AwesomeMath Summer Program, when utilizing this anonymous feedback channel, it was interesting to have three students request that scores not be given on their tests, but instead, they wanted more detailed feedback as to how they could improve each of their solutions. As they reach middle and high school, they need to start honing the very important skillset of social networking, which means highlighting the aspects of who they are that they wish to share. The Purple Comet is a team-based competition, so students have a community of peers with which they can compete to solve interesting problems. Each instructor brings their own story and experience that shapes the way they teach their classes. Through the next example, we provide an application of the pigeonhole principle in a branch of mathematical analysis, called Ergodic theory.1 Example 4 We choose 5 numbers from the interval (0,1). Probably the simplest is to use the following diagram and to notice that the combined area of two small squares in the left, a2 + b2, is equal to the area of the square c2 in the right part of the diagram. Then, you can provide question frameworks to get them started on the path of inquiry. There are n − 2 triangles in total. Exploration should, of course, be a shared experience. Students who don't have a group (maybe the number of students in your class isn't divisible by four) can take on rotation roles where they go from group to group and add value. If you don't already belong to a math league, a great first step is to create a math club in your school. Hence triangles FDC and EFB are congruent, and so BE = DC = 4. Polygonal numbers are a generalization of triangular numbers, allowing students to create and analyze various sequences. Now I think that math is not only problem solving, but also an art of asking math problems. We say that two cells of the 10 × 10 table are friendly if they have at least one common vertex. This leads to the next reward: resilience. On her adventure, she receives aid from helpers and mentors while she works through trials and challenges. Columns 1 and 3 should be much longer than columns 2 and 4. What are the objectives of the class? Since ∠ABC = ∠A'B'C′, it follows that AB and A'B′ are parallel. If we look closer, we can notice that the second magic square is exactly the same as the first one but rotated 90°. These peers don't necessarily have to be kids at their own school; they can expand to finding teammates, for example, at their local library or mathematics enrichment centers. Her understanding deepened and her love for the topic, in turn, grew. –Carl Jung Teens don't have control over many parts of their lives or the life skills to truly understand what all goes into planning things such as a school curriculum, or a job, or a household, or even just making a meal – they will sit in judgment on what is wrong or could have been done better because they have enough experience to compare, but not the depth of understanding to know the complexities. See Science, technology, engineering, mathematics Stottile, Frank, 17 Straight angles, 177 Strategies: efficacy of, 23; for engagement, 54–55; for goals, 45; Pigeonhole principle as, 228; for solving, 23 Struggle, 13, 98–99 Students: active creativity for, 54; with ADHD, 67; attitude for, 61; authenticity for, 22–24; in AwesomeMath Enrichment programs, 55; challenge for, 76–77, 150, 153–154, 162–163; choices for, 53–54; collaboration for, 107–108; confidence for, 98; control for, 65; conversations between, 92; discrete math for, 2–3; dissection time for, 239; encouragement for, 79–80; in The End of Average, 36; engagement with, 75–76; evaluation of, xvii; factorials for, 191–197; gains for, 62–63; goals for, 43–44; guessing by, 223; Impostor Syndrome for, 39, 64; individual, 36–37; information for, 5; inquiry by, 48–50; interaction between, 91; introspection by, 30–31; judgments for, 97; knowledge for, 27–30, 44–45; mastery by, xvii– xviii; movement for, 21; nice numbers for, 255; numbers for, 185; observation by, 149; pains for, 63–64; Pascal's triangle for, 245; peertutoring for, 95; Pigeonhole principle for, 227; planning for, 11–12; polygonal numbers for, 205; praise for, 95; problem-based curriculum for, 61–67; Pythagorean Theorem for, 213; relevance for, 13–14; retention of, 62; roleplaying by, 49; scale for, 17; schedules for, 1; selfassessment by, 23; sequences for, 221; skill gaps for, 70; skills for, 58; speakers for, 18; struggle for, 13, 98–99; student-centric learning, xviii, 37, 59, 62, 68; suggestion box for, 163; teachers and, 2, 92, 194; triangular numbers for, 199; upfront work for, 37; value propositions for, 62; Viviani's theorem for, 235; waiting for, 6. Then A = 1, B = 9, and C = 8. Find |A|. Following are some ideas for increasing range in a mathematics program: Logic problems. In the United States, founding father Benjamin Franklin also spent time constructing interesting variations. Unlike the traditional situations, when I expect the students to come to a solution (possibly with my help), I present a solution containing mistakes to a well-defined problem and throw the gauntlet down for them: Find the mistake in this solution. Example 3 Calculate 7!T7 − 3!S5. How can bumblebees fly? Focus their efforts to be more effective problem solvers. Hence, each display has 101 pieces and weighs 2020 g. Growing up in Mexico, I was not very interested in mathematics as most of it seemed like a lot of memorization, such as learning the multiplication tables. By the pigeonhole principle there exist two chosen numbers that are paired. What is x? Before collaboration can even begin, the groundwork of a kind space where all ideas are welcome must be cultivated and modeled. By taking risks and allowing for the vulnerability to make new discoveries, students can follow their curiosity, ask interesting questions, and work toward mastery. Math Olympiad for Elementary and Middle School (MOEMS). In most of my classes, there are worksheets or quizzes almost daily (most of these are in groups). Even rotating through your students' names to use in the problems is a fun way for them to feel connected – it doesn't always have to be Alice, Bob, and Carl. The outdated criterion of identifying top students through grades is flawed; it's evaluating someone's worth based on an outcome and not the process, which sets up situations where students avoid intellectual risks so they can maximize grades. doi=10.1.1.122.3025&rep=rep1&type=pdf. Students, to truly be educated, need to be just as interested in what doesn't work as what does work. Purple Comet! Math Meet 2019 contest, . On a typical semester, I work with anywhere around 10–25 students mostly on research teams in a wide variety of projects that range from math biology to data sciences. His passion for problem solving and mathematics teaching has extended to the following noteworthy accomplishments. Solution An equilateral triangle can be dissected into six, seven, or eight equilateral triangles, as shown in the figure below: The conclusion follows from an inductive argument. In particular, every positive integer can be represented in this form. Further, success means different things to different people. Thus, the first row with such property is the 48th row. Math education, or all education for that matter, needs to be human centric to prepare students for a world where critical thinking, talent, problem solving, and collaborative environments reign supreme. This is how you synthesize complex mathematics concepts during the lecture, either a recorded lecture if using a flipped classroom approach or during class, so that the information is retained based on an individual student's learning style. First, they learn that x2 = x*x is the area of a square of side length x. Be vulnerable and make mistakes in front of your child. Creativity requires bravery and resilience. Area of a polygon. In this way, luck can be better managed by keeping opportunities open and putting forth the effort to mitigate its negative effects. No warranty may be created or extended by sales representatives or written sales materials. Since the sum of the interior angles of the five-side polygon (i.e. pentagon) DEFGP is 540°, then d + e + f + g + α + 2β = 540°. A good leader will have the perspective to know the well-being and growth of the groups so as to set realistic goals in order to achieve their vision. Place the regular pentagon in relation to an equiangular pentagon, as shown. This works best when the problems are of various difficulty levels. There are five main reasons to enter math competitions: 1. A focus on collaborative problem solving is a great way to attract students to and prepare them for careers in math. You can run a leadership fire drill game, where the leader is the one holding an object and whenever you ring a bell, they have to pass the object to another table mate who then takes over the role with the goal of solving as many problems as they can during the class problem solving session. Draw a parallel line though A to BC. It's a constant process – relate, reflect, revise – and will soon just be a part of your everyday teaching. DEFINITIONS Regular polygon. There are two solutions provided that highlight different skills sets. A couple of great websites to explore are Cut the Knot1 and the National Library of Virtual Manipulatives2 for online puzzles and games. In mathematics education, it provides students with both the short view, what they need to accomplish to be successful in school, and the long view, what they need to learn to be successful at whatever occupation they choose. Therefore, it's not only important to facilitate learning but also to help develop the students' mathematical compass so that they can begin to self-diagnose issues and remain authentic to the process. People have incorporated Roman numerals in artifacts, inscriptions on buildings, clocks, chapters in books, and chemistry as well. Some other small primitive solutions, given as edges (a,b,c) − face diagonals (d,e,f) are: (85,132,720) – (157,725,732) (140,480,693) – (500,707,843) (160,231,792) – (281,808,825) (187,1020,1584) – (1037,1595,1884) (195,748,6336) – (773,6339,6380) (240,252,275) – (348,365,373) (429,880,2340) – (979,2379,2500) (495,4888,8160) – (4913,8175,9512) A perfect box is an Euler brick whose space diagonal also has integer length. Example 4 In how many ways can you get the number 100 by adding some consecutive integers between 1 and 99 inclusive? 15 Equilateral versus Equiangular OVERVIEW “Equilateral” and “equiangular” are two special types of polygons with some interesting qualities. They can curate their own entertainment, decide what fashions look best for their tastes, and build, customize their electronics, and choose their identities based on what feels natural to them. This work aims to broaden students' view of mathematics and better prepare them for possible participation in various mathematical competitions. c. CHAPTER 15 Sequences Learning Objectives Students will be able to identify and extend arithmetic and geometric sequences. Usually there is more than one way to solve a toothpick math problem. What is the least common multiple of 6, 8, 24, and 30? Let us denote such a distance by D. Figure 7.4 According to Bolzano's theorem, there exists a point M in which t... This approach, however, requires facilitators, helpers, and guides along the way so that each student can recognize their value and be their best version. Dr. Emily Herzig: Encouraging conversations between students during class is a cornerstone of my approach to teaching. Solution Note that Thus, is, n = 5. Torricelli was the first to solve the three-point case (the problem became known as the Fermat-Torricelli problem). A math assessment test can help teachers determine any skill gaps a student may have so that they can be addressed and the gaps filled. Finally, the number of spots leaves a remainder of 6 when divided by 9. Wacław Franciszek Sierpiński was a Polish mathematician. Solution a1 = (1/1), a2 = (1/2), a3 = (1/3), a4 = (1/4), a5 = (1/5). People feel that creativity just happens, when in actuality it is the product of trial and error – requiring the resilience to persevere. The International Olympiad in Informatics (IOI). Here are some value words to get you started: Kindness Collaboration Creativity Honesty Innovation Excellence Empathy/compassion Community Independence Positivity Motivation Respect Perseverance Flexibility Focus Curious Commitment Determination If students feel valued, listened to, and respected, they will put their best foot forward to work toward a common goal. Again, it's important to stress that the end goal of these competitions is not a winning score, which is certainly a positive byproduct, but its exposure to interesting problems in varied topic areas as well as the comradery and “coopertition” that competing can bring to a group. An example would be to introduce the concept of magic squares and have each student make their own and share with the class. The Art of Problem Solving4 organization hosts many of these forums where students can come together and discuss competitions, problems, and find resources. Also, 7k ≤ 2009 is equivalent to k ≤ 287, so |B| = 281. Solution Since we deduce that That Hence, n = 7. Let's say that the threshold is crossed, and our hero must traverse many twists and turns when, all of a sudden, the cave splits into two paths. However, taking students who are terrified of mathematics and having them learn to use math as a tool to make something happen in real life is my newfound passion and is incredibly rewarding. In a triangle, the nine-point circle is tangent to all circles tangent to the side lines of that triangle. One empties in 4 minutes, the other in 7 minutes. Note that this gives a carry of one out of the tens column. Thus, the numbers are 16, 17, 18, 19, 20. The sum of the denominators in the equations above are called “nice numbers.” Example 1 Is 9 a nice number? Many times, the students will want their teacher to tell them if their work is correct, if the problem will be on a test, and/or whether or not they absolutely need to know the information. By recording your process in a notebook, you can always go back to the problems you have started, and as your skills grow and your insights deepen, you may be able to rework and/or solve them – it all requires practice, patience, and persistence to keep going! It also requires quite a bit of time solving problems written by others, with the understanding that not all good problem solvers are necessarily good problem authors at the beginning. Can you share your ideas with others and, better yet, the story along with your ideas? Play is an integral part of life. Example 3 Prove that among any six numbers from the set {1,2, ...,10} there is one that is divisible by another one. There are a number of games in Chapter 7, but one that is quick, easy, and conducive to active learning is Buzz. Hence, k = 21. Hence, by the pigeonhole principle, at least one of them appears at least 17 times. Combinatorics is the art of counting minus the counting! This topic introduces the “art” by considering elementary cases dealing with finite sets. What people don't see is the countless hours spent drawing, sketching, and painting that shapes the artist's talent so that they can produce beautiful compositions. Arguably, this process is also what makes math such an enticing subject. Geometric progression. The following relation holds: |A∪B∪C| = |A| + |B| + |C| − |A∩B| − |B∩C| − |C∩A| + |A∩B∩C| The argument is similar to those presented above for two sets: Exclude the elements of intersections A∩B, B∩C, and C∩A and then include the elements of A∩B∩C. See Students Standardized testing, 60, 63, 69 Stankova, Zvezdelina, 16 Steinhaus, H., 165 STEM. They cannot both be 100°. What three qualities of the applicant would shine through when read by the hiring manager? Also, in one of the columns, two A's add up to T = 9. A patterns of horizontal and vertical lines, usually forming squares. It could be an image on a PowerPoint presentation or it could be an actual chess board. The diagonals of a rhombus are angle bisectors of the respective angles of the rhombus. Only one subset, {E, M, O, W}, is favorable, so the probability is Solution 3: Yet another solution is based on what each of Meow's paws has pressed, in any particular order. It is called the Bolyai-Gerwien theorem. Note that the number at the end of the number is the number of times we can divide this number by 10. Then find the lattice points of intersection. The area of the polygon is equal to ½d1 × a + ... + ½dn × a, where a is a side length of the polygon. The next term in the sequence is 120. It helps students feel more comfortable making mistakes and by doing so, learning by experimentation. 29 = 2 × 10 + 9, we showed before that 10 is nice; thus, 29 is also nice. TEACHER GAINS Teacher efficacy Deeper learning Retention When using a problem-based approach, teachers have noted improvement in their own mathematics skills and teaching abilities. Thus, the perimeter of the polygon is 2 × 8 + 2 × 12 + 5 × 4 = 16 + 24 + 20 = 60.1 Students are allowed to be transported by a musical symphony before understanding individual notes, so why not provide the symphony of mathematics and introduce students to its wonders and challenges? Therefore, the above sum is equal to: Since 12 + 22 + ... + 202 = (20 × 21 × 41)/6 and 1 + 2 + ... + 20 = (20 × 21)/6, we find that the above expression is equal to: Notes 1. Be proactive and capture attention. Some students know the steps that need to be taken to find the square root of a number, but very few have understood why you would need to do so. S. All of this is part of the journey, so don't let it discourage you from the process. Here is an example of how we can dissect a triangle into a square. In calculus class, much of our time was spent working in groups, and the discussion of problems got very intense at times. Be excited about the process. Wait until you learn a subject, like geometry, in isolation before you have the ability to learn how it connects and contributes to other areas such as algebra, engineering, art, science, etc. Post Reflections After the demonstration, Jenny and I decided to dig up some interesting facts about the history of the problem. It is not enough, for example, to note that the year 2019 is an odd number. SECTION III Full Units In this final section, you are provided with full units that range in difficulty from the easiest, “Angles and Triangles” to the hardest, “Nice Numbers.” That said, each unit has problems that are accessible along with problems that will provide a significant challenge for high-level students. We've seen first-hand the leaps in skills, growth of curiosity, and joy of problem solving that arises when individuals are immersed in a kind, collaborative, and challenging environment where students create positive life-long memories and form valuable friendships. Motivation and accountability. Solve the following equations in positive integers. 30 = 2 × 11 + 8, we showed before that 11 is nice; thus, 30 is also nice. This, of course, can be the goal of any subject; however, mathematics is an exceedingly efficient way to learn the inductive and deductive reasoning necessary to make smart decisions and solve problems. Many interesting and delectable meals have been invented by people who are constrained by budget, choices, or geography. Brazil, M., Thomas, D., Graham, R., and Zachariasen, M. 5. Expose students to “real-world problem solving.” Working in teams and learning to collaborate. Because I've learned a lot of mathematical concepts through problem solving, I've never had to ask myself, “Where are we going to use this?” It was already clear that most concepts I'm learning explain something that I previously thought is a surprising phenomenon, but which can be very well explained using some more advanced concepts or theories. A polygon with all sides the same length and all angles the same measure. Let A and B be two sets such that |A∪B| = 20, |A| = 16, and |B| = 17. For example, some students in middle and high school may still struggle with understanding fractions that would be a critical gap to have assessed. This is a huge skill going forward in life. 16. To provide the curriculum plans and interesting problems that allow educators to successfully train their students to think with a problem-solving mindset. PROBLEMS Directions: Prove these statements to be true. This top-down approach to education does not benefit today's students, or in Mr. Rose's vernacular, an education for the average student is designed for no one. You'll note that some of the pains stay the same for each group, but this is especially so for the gains since in the end, students, teachers, and parents alike are seeking a good education. There are three different levels of AMC tests: AMC 8, AMC 10, and AMC 12. One important goal of any educator is to tap the internal motivation of their students so that they can efficiently and effectively learn the material necessary during the school year. PBL needs to have meaningful problems where students enjoy the process and want to work toward mastery both in the group and on their own. The sum of distances from a point to the side lines of an equilateral polygon does not depend on the point and an invariant of the polygon. It follows that the largest square has an area that is 8 times as large as the shaded region. I am heavily involved in directing undergraduate research. We can check that a = b = c = 2 and d = 3 is the only solution. When do you think you could apply this concept in the future? Fibonacci Numbers To obtain the Fibonacci numbers, we sum the entries of leftaligned Pascal triangle: The first eight numbers in Fibonacci sequence are 1, 1, 2, 3, 5, 8, 13, 21, found in Pascal's triangle. Also present the pieces. Peers are also critical in this endeavor so that ideas can be discussed and approaches explored. Sometimes, however, motivation needs a jump start. Still others may only wish to acquire the critical thinking a problem-solving curriculum can provide to pursue careers outside of mathematics. four consecutive numbers c. TEACHER PAINS Time intensive Carving out collaborative problem-solving time into a single class period along with a short lecture of the topic is difficult. Test scores Knowing how to approach problems, deconstruct them, think critically, and imbibe foundational material increases student test scores. Then we found the picture I was thinking about in the book What Is Mathematics: How lucky I was to have forgotten this picture, but not – what mathematics is! Probably, the experimental physicist Arthur Stanley Eddington is right to think that proof is an idol before whom the pure mathematician tortures himself – but they forget to add that it is the tortures and the successful debugging process that let us experience the pure joy at the end.8 Further Reading Gander, M.J., Santugini, K., and Steiner, A. The leader also needs to be aware that it is a team effort, not their moment of glory, and as such, the team needs to feel that they are listened to and their opinions are respected. AMC 10, which is for students in 10th grade and below. It's similar to someone being a natural artist. Creativity is a free-thinking process, and the mind will follow various thought paths in order to come up with new ideas; however, this may lead a problem-solving team astray if the creativity isn't somewhat bounded with certain constraints, e.g., time limits, mathematical parameters, etc. There are so many competitions from which to choose that a teacher can find the contests that best fit their class schedule and approach. The final pain listed for parents is the importance of grades. Solution: Start both clocks at the same time. However, as stated above, the hero is not alone, and she has helpers and mentors who can guide her along her path. Integers without any positive common factor other than 1 and −1, or if their greatest common divisor is 1. See also Teacher inspiration B Bach, Johann Sebastian, 51 Beethoven, Ludwig van, 51 Behavior, 49, 93 Benjamin, Arthur, 155 Biographies, 18 Boards, 98–99 Bolyai, F., 239 Bolyai-Gerwien theorem, 239–243 Bolzano's theorem, 138 Bottoms-up approach, 36–37 Brocard-Ramanujan problems, 194 C Calculators, 76 Campbell, Joseph, 38–39 Cantor, Georg, 48 Capital, 111, 113–114 Cargo cult science, 21–22 Carnegie Mellon University, xvii–xviii Case studies: on AwesomeMath Enrichment programs, 5; on education, 2; on enrichment schools, 119; on geometry, 213; on information, 14; on math circles, 17; on MATHCOUNTS teams, 97; on research, 8; square roots in, 22; on teachers, 58; on teaching assistants, 80 Catalan's identity, 88 Cayley, Arthur, 160 Chalkboards, 98–99 Challenge: collaboration and, 42; in community outreach, 92–93; Conrad Challenge, 107; continuous, 60, 63, 68; discovery and, 21; engagement from, 15, 80; M3 Challenge, 105; Paul Erdös International Math Challenge, 104–105; Pythagorean Theorem for, 33; skill gaps as, 66; for students, 76–77, 150, 153–154, 162–163; for teachers, 10 Chemistry, 107 Chess, 170–171 Choices, 53–54 Chunking, 80–81 Circles, 219 Clarity, 42–43, 51 Classrooms: collaboration in, 112; environment for, 79–81; as flipped, 37–38, 58, 101; management of, 93–96 Coaching. Therefore, b. The provided examples will bring to life the important concepts and takeaways for the lesson. The “Factorials!” unit includes problems that are interesting but easy to miscalculate. Property of the area of a triangle: The product of the length of a side by its corresponding height is independent of the choice of the side. Does your strategy work? Some are complementary and some are not. When coaching math teams, one way Kathy set up the room was to have students at various “level” tables. Having a communication channel that allows for anonymity allows students the freedom to provide constructive feedback. It's critical that students are surrounded by people who want to see them improve and motivate with positive energy. Making life-long friends they will encounter again in college, scientific conferences, and when they travel to Stockholm to pick up their Nobel prize. That means having a well-defined purpose. Then, A1OA2, A2OA3, A3OA4, A4OA5, and A5OA6 are equal triangles. By seeing their value and giving them autonomy, you raise the expectations and, in turn, show respect for their ability, which provides tremendous motivation. Students need to fight against the temptation that the world is their personal echo chamber for their latest thoughts and feelings. CHAPTER 20 Nice Numbers Learning Objectives Add reciprocals of positive integers to represent any rational number. When students are viewed as a collection of strengths rather than a collection of weaknesses that need to be fixed, they will see the value they can add working on a fun, new challenge! This value will shine through even more when they collaborate with a group of people working together to solve a difficult problem. At the AwesomeMath Summer Program, the concept of identity capital is addressed right away at opening ceremonies. Students will do much better with these problems if they're paired with accountabilibuddies. – Titu and Alina My heartfelt appreciation goes to my closest community, my family, for their support, advice, and contributions to this effort. Example 6 What is the greatest number of bishops we can place on a regular 8 × 8 chess board without any two attacking each other? Furthermore, they aren't being prepared to face the current challenges in today's workforce, which values innovation, leadership, collaboration, resilience, and critical thinking. You can challenge yourself to move up levels and share your experiences with peers – plus, there is no fear of losing, whereas in mathematics, there is fear. A messy problem that your community can help polish up is just as valuable, if not more so. Theorem (Viviani theorem, second form). Building things? It follows that g2 = a2 + b2 + c2, implying . With patience and the proper questions, they can figure out for themselves if their solution is true. Leaders must model the integrity that contributes to this positive environment. Pacing requirements. Those who prize efficiency love to solve problems quickly, so they can work on more problems! They follow a methodical and linear progression to problem solving and tend to have incredible memories. While those skills are certainly useful for efficiently carrying out the basic mechanics of solving problems, it is equally important that students are able to formulate and interpret more complex problems and work with their colleagues to develop and execute problem-solving strategies. Patience and diplomacy. This is because you are still improving the more chess games you lose, but that is a stretch of the definition. The mathematics of science. An angle measuring 180°. Looking at the tens column, the ones digit of X + Y + Z + 1 is Y. This is also a technique when creating a resume or writing a cover letter. In some cases, especially parents who are already in science, technology, engineering, mathematics (STEM) fields, the parent may actually have more discrete math training than the teacher. If the ladder is lowered so that its top is 37.5 ft from the ground, the distance from its bottom to the wall increases by 13.5 ft. Yoko Ogawa, The Housekeeper and the Professor That means setting the tone from the beginning that mathematics is not something to be feared and that mistakes are positive and necessary for growth. Although I never set out an objective of being like these students or even becoming a “mathlete,” seeing a spark in my father's eyes every time I explained to him how I'd solved a puzzle or a problem definitely had a huge impact on my further interest for mathematics. Frank Sottile, “The Shape of Space,” lecture at the University of Texas, Dallas, November 4, 2017. Students and staff travel from around the world to learn exemplary mathematics in a community that connects inside and outside the classroom. For some positive integers m and n the following displays use the same number of pieces: an m × (n + 2) rectangle with a 8 × 8 square removed from its interior; an (m − 2) × n rectangle with a 4 × 4 square removed from its interior; an (m + 4) × (n − 6) rectangle with a 2 × 2 square removed from its interior. Parents know their own kids and can question them about their approach to the problem at hand. And, as always, how much people want to choose themselves versus what is chosen for them is a spectrum. When students are working on collaboration teams, ensure that the student leader for each group also changes. Without loss of generality, we may assume that both paths lie in the square; applying the Bolzano's theorem, we know that there exists a point M in which two paths meet (Figure 7.4). How can you maximize mathematical thinking in the classroom and increase that connection? Sum of entries of nth row of Pascal's triangle is equal to 2n. Conrad Challenge. The equation 180° − (360°/n) = 160° can be written as 180° − 160° = 360°/n, which implies 20° = 360°/n, yielding n = 18°. When going through the mini- and full units, it is important to take advantage of the Cornell note-taking method discussed in Chapter 6 through the relate, reflect, and revise process. Yet another way to prove Pythagorean theorem is based on the leg theorem. Solution Because 18 and 2019 are both divisible by 3; so is pqr. Using a calculator, answer the following questions. Did they understand how the lesson relates to other areas of mathematics? Experiment with simple cases. Provide an Active Learning Environment With problem-based learning, a lot can be going on as students interact with each other, present problems on the board, ask for help from the teacher, and continuously question. Published by Jossey-Bass A Wiley Brand 111 River St, Hoboken, NJ 07030 www.josseybass.com No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400, fax 978-646-8600, or on the Web at www.copyright.com. (or: x − 1, x, x + 1). This means, that as the facilitator, you need to be knowledgeable, encouraging, and most of all, patient. Since the AwesomeMath community is international, she must meld a diverse background of individuals into a thriving learning environment. Therefore EF = 3, implying that EF = FD = 3. Separating into pieces. Adventures aren't always appealing at the outset. His recommended exercise to reclaim that power, which has worked well with our students who have math anxiety, is to have them create a four-column table under a heading where they “Name the Fear.” So, for example, they may create a heading that says, “Solving a problem on the board.” The four columns would address the heading as follows: 1. How would you connect your problem-solving approach with the lesson? Therefore, a + b + c + d + e + f + g = 900° − 2(α + β) = 900° − 2(180°) = 540°. Dr. Andreescu was presented with a gift from a colleague who had collected some of the problems that he had written when he was still in middle school and high school in Romania, and upon first glance, Dr. Andreescu was critical of the caliber of the problems. Again, from the Pythagorean theorem, d2

you, the reader, to fish for good problems as well as be able to write your own. We found numerous sources. Since 33 of the numbers are divisible by 2 × 3 × 5 = 30, applying the inclusion-exclusion principle we get Magic Squares OVERVIEW Magic squares are square grids with a special arrangement of numbers in them. In fact, our preferred term is the portmanteau of cooperation and competition, coopertition. Everyone has heard that Albert Einstein wasn't a very good student – it makes him vulnerable and achievement seem more attainable. When a student is struggling and exclaims, “I'm not good at math!” you can explain that output is a function of effort: “If you put in the time, I know you can do this!” and provide hints, encouragement, and connection (relatedness). Thus, y is not an integer. The Cornell note-taking system was created by Walter Pauk, an education professor at Cornell University, as a framework for students to organize, synthesize, and better learn from their notes. By the pigeonhole principle there exist two numbers such that one is divisible by another one. The fundamental theorem of proportional segments: The corresponding segments on the legs of an angle cut out by parallel lines are in the same proportion. Why Is a Collaborative ProblemBased Approach Worthwhile? The Sage on the Stage approach is lecturing from a central point to your audience, but with design thinking, you can mix it up! Keeping a sense of discovery and newness in mathematics is critical, and problem solving, with interesting challenges, ensures that the thrill stays alive. The world changes at such a pace that if individuals and organizations don't know how to think and pivot during their journeys, they will be left behind. In the above definition, we use the letter S to describe an entire sequence. Understand the Value of Mistakes There are wonderful ways to guide and facilitate learning that don't judge or evaluate, because you want your students to make occasional mistakes. Meow's paws land on four different keys. Example for the first four numbers n y Δ#s 1 1 1 2 4 3 3 10 6 4 20 10 Sum of Δ#s 20 n = number, y = tetrahedral number 3. Thus, (9! + 11!)/(8!x + 10!) = 9!(111)/8!(90 + x) = (9 × 111)/(90 + x) = 9. Rather, we encourage you to expand on the ideas presented to create a curriculum that works for your style and the students who make up your classes. And, maybe most importantly, we learned because we talked about the concepts, while working with others, and we learned because we teach! Collaborative work helps emulate this environment and helps students that know a bit more really secure the concepts by trying to teach others. Solution Let the first number be x and the second number be x + 1. Will he give up and say, “Well, I guess I'm bad at video games, so I'll stop playing”? For additional practice, use the books on the reading list.Contents:Preface Acknowledgments Abbreviations and Notation 1. Students must learn how to work in both types of situations, and much of that is knowing themselves well and knowing what value they add in different types of group settings. The next number in the sequence is 17. The interesting property of Fermat numbers states that lemniscates (any of several figure-eight or ∞-shaped curves) could be divided into d equal parts with ruler and compass if d = 2rp1, …,ps, where r is a nonnegative integer and p1, …,ps are distinct Fermat primes.2 In this figure, you can find the position of Fermat primes in Pascal's triangle. Solution: 3. The quantitative measure, in square units of the interior surface of a two-dimensional figure. We recognize that the denominators are powers of 2. Please note that some of the lessons can be split into 10-minute units given over a two-day period. All angles are equal. Puzzles and simulations are individual pursuits, but students can parallel play and talk to each other while doing them. Their story begins 4000 years ago in China, where, according to legend, a turtle crept out of the Yellow River. Ergodic theory is a branch of mathematics that studies dynamical systems with an invariant measure and related problems. We have 5k ≤ 2009 is equivalent to k ≤ 401; hence, |A| = 401. By providing the curriculum, questions, techniques, and solutions for every educator, the pains detailed above can be relieved and, in some cases, cured. Consider the set A = {x|x is a positive integer and 345 < 3x < 3210}. The segment connecting the midpoints of the diagonals of a trapezoid is parallel to and has length half of the difference of the bases. gander/Preprints/BDM56GanderE.pdf. Sometimes, they are in your favor, and sometimes they are not. Valery can at most chose 11 divisors. Model the behavior. Find the four consecutive integers that add up to 50. What would you do differently? Georg Cantor Working toward mastery requires vulnerability, and that means asking for help. Have access to short lessons on days when you have other pacing requirements to satisfy. We can assume that a ≤ b ≤ c. Beyond being connected with multiple math groups, Kathy is also a part of a network that includes parents, teachers, and students, such as AwesomeMath parents, students, alumni Purple Comet supervisors/teachers Davidson Young Scholars, parents, and alumni Mathematics organizations Homeschool groups Alina Andreescu was born and raised in Romania, at a time when mathematics education was exceptionally strong. Even if you can solve a problem, do read the solutions. As an educator, it is imperative to create a culture of curiosity, and you can do that by training students to continually question. 2, 3, 5, 7, 11, 13, … 4. This shows that the right-hand side quantity remains constant and does not depend on the choice of P. GRACE Ego is the hubris or arrogance that you know all and it decreases your flexibility, whereas on the flip side grace is the humility to understand that you are fallible. and collaborative. Fifteen years later, these percentages have shifted to closer to 40% inspirational stories, 30% letting students present, and 30% lecturing, and I'm continuing to evolve in this overall direction. Creativity can inspire innovation, and a good leader will help the team harness focus so that both can take place. The following list comprises traits that make up good leaders, and each item has two qualities listed. The next number in the sequence is 13. In lieu of pointing out mistakes, it is more beneficial to ask questions about the truth of their responses: Why do you think this solution works? Following the traits will be a suggestion for how to use the curriculum units included in the book to guide your students to be good leaders: Confidence and humility. The next term in the sequence is (7 × 8)/2 = 28. Example 5 We mark 201 points in a 10 × 10 square. This discrete mathematics field involves counting and probability. If n is an integer such that find. This produces two sets of congruent angles (β and θ) equal angles and four supplementary angles. The struggle of working through a problem is so important, and yet you don't want your student to feel embarrassed or humiliated. There are six possible scores throwing one die: 1, 2, 3, 4, 5, 6. Therefore, you would take the opposite cave. These are the questions that were sent to the AwesomeMath team and they were instructed to answer as many as they liked: What were your own school experiences like in your country that contributed to your love of problem solving? These are all representations, since 2 × 99 = 21 × 32 × 11 has (1 + 1) (2 + 1)(1 + 1) = 12 divisors different from 1. Rigor is both the result of critical thinking and creativity as well as the discipline to learn the foundational concepts necessary to be successful. A PBL curriculum can be challenging for them, since it may be different from their own education experiences. SECTION II Teaching Problem Solving In this section, we will provide teachers with the strategies and tools for guiding innovative problem solvers who can work independently and think deeply. What activity(s) would make the student lose all track of time? Cargo cults are part of religious practices where tribal cultures who have not been exposed to industry or technology attempt to mimic the motions necessary to receive the cargo, or bounty. IDENTITY CAPITAL We all only have a finite amount of time, and there are investments that must be made of that time. Example 3 Are 3 and 5 nice numbers? A problem-based curriculum can be a boon for parents since their children will be better prepared for the remainder of their middle and high school years as well as gain the thinking skills necessary to excel in college and jobs. In our experience at AwesomeMath, many parents are seeking a map of engagement, in other words, a way to work with their child at home so they get the most out of their education. StudentStudents who are at the center of their own learning centric experience are more willing to participate and see the value in the lesson. Following one another in uninterrupted succession or order; successive: six consecutive numbers, such as 5, 6, 7, 8, 9, 10. Here are a few example diagrams from previous students: Student A. In the figure, students have a “cue column” and a “note-taking column” to record information. Each student can add value in their own way. On the history of the Euclidean Steiner tree problem. It really reveals who is persistent and creative. Meaningful When the problems connect to real life for the students problems and they see the use beyond the class, they are more curious about the material and intrinsically motivated. 24 = 2 × 11 + 2, 11 = 2 + 3 + 6, 1/2 + 1/3 + 1/6 = 1; therefore, 24 is also nice. The teacher asks Valery to choose some of positive divisors of 200910 such that no chosen number divides another. Students, through rigor and range, grow their courage for trying new things as well as hone their creativity and resilience through constraints. See also Resources Teamwork, 95 Teenagers, 113 Teleology, 47–56 Testing: approaches in, 45; information and, 9; scores in, 63, 68; skills and, 36; standardized, 60, 63, 69 Tetrahedral numbers, 203, 249 Theorems: Bolyai-Gerwien theorem, 239–243; Bolzano's theorem, 138; Ergodic theory, 229, 233; Galois theory, 145; Pick's theorem, 165–168; Pythagorean Theorem, 32–33, 77, 213–219, 239–240; Theorem of Thales, 90; theory, 191, 228, 245, 253; Viviani's theorem, 168, 235–238. I'm trying to select problems that are neither too easy nor too hard. Then x(x + 2) = 45. Then: x + (x + 1) = 33, which implies that x = 16. It's an easier mental leap to begin to learn binary systems (base 2) that are integral in today's digital world. See US National Chemistry Olympiad V Value propositions: for parents, 67; for students, 62; for teachers, 59 Venn diagrams, 20, 30–33 Vertices, 202, 205–206 Vision, 111 Viviani, Vincenzo, 239 Viviani's theorem, 168, 235–238 Volume, 172–173 Vulnerability, 49 W Waiting, for students, 6 What if mindsets, 78 Wilson prime, 191, 193–194 Work, 6, 37 World War II, 21 Y Yotov, Mirroslav, 19, 128–131 YouTube, 145–146 Z Zeros, 196–197 WILEY END USER LICENSE AGREEMENT Go to www.wiley.com/go/eula to access Wiley’s ebook EULA. As Dr. Andreescu states when he is working with teachers in his graduate level education courses, “Mathematics is not a spectator sport, it is a team game,” and that's why having students work in groups will make the learning process much more effective. For further problems and training resources that can be utilized in problem-based learning classrooms, visit ! CHAPTER 9 Angles and Triangles Learning Objectives Students can identify angles and types of triangles. Finding your Venn is all about determining what puts you into a state of flow. Therefore, the requested area is 8 × 7 = 56.4 Reason, logic, facts, conjectures, theories – they all roll together to create tomorrow's thinkers! Students who know how to evaluate research studies, statistics, medical reports, business plans, and so on can look at the data, ask questions, and find truth. For example, there can be a time constraint that diminishes the effectiveness of a problem-based curriculum. Agile software development refers to a group of software development methodologies based on iterative development, where requirements and solutions evolve through collaboration between self-organizing cross-functional teams. More times than not, students will veer off path because they haven't clearly read the problem. Prove that there is only one Pythagorean triangle whose side lengths are consecutive integers. It is the very process of starting with a wrong idea, becoming aware of the error in reasoning, and finally, succeeding to correct it, which I have kept reproducing when teaching that course. Analogously, triangle GBC is isosceles. The lines AB and CD intersecting at a right angle are called perpendicular. Vision requires the ability to motivate your collaboration team. Remark Finding the positive integer solutions to the equation m! + 1 = n2 is still an open problem (it is indeed the Brocard-Ramanujan problem). Du, D.Z., Hwong, F.K., Song, G.D., and Ting, G.Y. (1987). See International Mathematics Olympiad Impostor Syndrome, 39, 64 Inclusion, 111–112 Independence, 43 Individuals, 36–37 Inductive reasoning, 34 Inequalities, 89 Inexperience, 63, 69 Information: case studies on, 14; for students, 5; testing and, 9; understanding and, 13, 23 Innate talent, 99 Innovation: collaboration and, 76–77; creativity and, 109–110; rewards for, 42 Inquiry, 48–50 Inspiration: for learning environment, 44–46; from logic problems, 8. 4.5 6. E. Let α be the size of (n − 2) interior angles. The “Pick's Theorem” mini-unit lets students solve problems using Pick's theorem, but they can also use geometry instead. Find permutations. Show the algorithm for adding, subtracting, multiplying, and dividing fractions. I understand this question as, “Is a collaborative problem-solving approach worthwhile?” My answer is yes, it is worthwhile. The total area of the chocolate wrapping paper in the first box is A1 = 8 × 4π(d/2)2 = 12πd2, while the area of the chocolate wrapping paper in the second box is A2 = 4πd2. These exercises are mostly done in groups, with me as the facilitator making sure everyone in each group has a voice and their voice is heard. This is the only limitation, so for any odd length of side a we can find two consecutive integers b = (a2 − 1)/2 and c = (a2 + 1)/2, so that a, b, and c form a Pythagorean triangle. 149, 150, 151. You're Reading a Free Preview Pages 31 to 44 are not shown in this preview. This unit advances the skills from the “Triangular Numbers” unit, Chapter 12, including the construction of sequences and their relations. The second number is equal to the number of vertices of the polygon. Thus, the distances between the corresponding sides of the two polygons remain constant. Everybody decent at mathematics is encouraged (sometimes forced) by their teachers to participate in the first rounds of the Romanian Mathematical Olympiad. Remove four toothpicks to obtain five congruent squares. . Arithmetic progression. This book is a must-read for parents and educators in all subject areas who wish to develop their students’ creative and critical thinking skills.” Jaime Smith, Founder and CEO of OnlineG3.com “The book is an excellent source for educators interested in problembased learning through student-centric approach. In the figure below, lines L1 and L2 are parallel. So, 125! has 31 zeros at the end. Clear. Students are exposed to natural-language processing in this engaging linguistic problem-solving competition. For example, (1) 1/1 = 1 (4) 1/2 + 1/2 = 1 (6) 1/2 + 1/3 + 1/6 = 1. Otherwise, the sum of angles will be greater than 180°. “US College Dropout Rates and Dropout Statistics,” College Atlas (June 29, 2018), . The teacher is a facilitator and guide during this process, circulating through the room to offer hints and filling in knowledge gaps. This is an incredibly difficult thing to do, especially for some students, and even for some teachers! Presenting, processing information, and doing arithmetic in front of an audience of peers is nerve-racking. (Answer: There are hundreds of proofs, both geometric and algebraic; check out the full unit in Chapter 14, “Pythagorean Theorem Revisited,” to learn more.) Another method for out-of-the-box thinking is to add constraints to the box (back to the concept of “Resilience Is Born Through Creativity,” mentioned in Section 1.3, where we describe how creativity is stoked through constraints, e.g., living on a tight budget). What truly makes a problem-based learning curriculum come to life is when students organically form their own problemsolving groups and collaborate. There is an old joke: When I die, I want the people I did group projects with to lower me into my grave, so they can let me down one last time. They want to understand the problem. One way is to understand its origins. In today's culture, you can be a star athlete and take pride in your work and accomplishments, yet star mathletes need to be humble and keep their accomplishments hidden so that others don't feel less than. Solution This problem requires a subtle choosing of holes. However, the broadcast approach is based on the fallacy that the communication is being received and understood. Young students have so many unknowns ahead of them in life. D.-Z. So one possible solution is: x=1 y=4 z=7 SHOW RELENTLESS CURIOSITY Good problems start with good questions, and that means being relentlessly curious. Eventually, I have them solve problems and write up their solutions as an assessment. As the name suggests, triangular numbers can be visualized as a triangle of points. Solutions to Advanced Problems Glossary Further Reading IndexDescargar PDF Descargar EPUB Descargar RARBuscadorCategorías Something went wrong. Further, for many students, math is a negative word, and they can easily fall prey to either: “I'm not good at math.” “When will I ever need this?” This is especially true in cultures where it's okay to completely discount the importance of mathematics, and yet, so much of their future depends on mathematical literacy. Then ∠PAB is half of this circle and has 180°. If two sides are taken in a triangle, that are proportional to two corresponding sides in another triangle, and the angles included between these sides have the same measure, then the triangles are similar. Most often, I am attracted by the “standing” of the problem in the body of mathematics involved. Identify nice numbers. Time spent on our financial well-being Physical capital. It means striving for civility and respect in every and all interactions. The only equiangular triangle is the equilateral triangle. If the plane figure F has diameter d, then there is a regular hexagon of height d that contains F. Note that 99 = 7 × 14 + 1, so we need 7 holes. Without loss of generality, assume that r = 3. In the subtraction PURPLE − COMET = MEET, each distinct letter represents a distinct decimal digit, and no leading digit is 0. Prove that there are two numbers a and b (a < b) such that 0 < (b − a) < ¼. 15. Example 3 Prove that an equilateral triangle can be triangulated into n equilateral triangles. As we get older and bound by the way things should be, we get out of practice asking questions. The next letter would be Y. After sharing the whole experience and presenting a rigorous proof to Jenny, she proposed a more natural proof – namely, using a soap film. Square roots take off the training wheels of arithmetic and lead young mathematicians to more abstract mathematics and algebra. Therefore, d ≥ 1 + c. a. You're Reading a Free Preview Pages 50 to 51 are not shown in this preview. That means presenting the student with problems instead of exercises. We are given that Thus, we find that (2x + 17) = 231. Andreescu and Kisacanin, Math Leads for Mathletes, Book 1. Is there a focused and discernable direction apparent in the statement? This proves that both 17 and 18 are nice numbers. There is a huge difference between understanding how something works and deriving a solution on your own. Flexibility and creativity are required when working with students of varied levels and abilities, so this section seeks to provide a well-conceived plan, along with ideas from our teaching and learning community to inspire new ideas and methods. Titu Andreescu and Branislav Kisacanin, Math Leads for Mathletes, Book 2, page 14, part of Fun Sequences (Providence, RI: American Mathematical Society, 2018). The Europeans continued to utilize this system until the 1600s. Here is a complex problem that is sure to connect with the kids, at least with the kids who have cats! A kitty named Meow jumps on a computer keyboard that only has 26 keys corresponding to the letters of the English alphabet. If students aren't asking questions, then lead with questions you think should be asked for a particular problem. As their name suggests, they represent the number of dots needed to make pyramids with triangular bases. Through certain activities or discussing ideas or both? Find N − n. This following problem is an enticing visual that promotes class discussion and inquiry: What could be the next term in the following series of objects? Many of the problem-based materials supplied to teachers today are incomplete and don't provide the necessary support to make this approach attractive to use. Marilyn Monroe was born in 1926 and died in 1962. The side lengths of an equiangular hexagon are labeled, consecutively, a1, a2, a3, a4, a5, a6. It's not about solving the problem quickly as much as creating synergy to truly understand the depth and breadth of the material and respect each other's role. Show that the height h of a right triangle is equal to the geometric mean of the projections of the legs onto the hypotenuse, m and n, i.e. h2 = mn. Thus, all integers n = 2p where p is a prime are good. Assume we have six points in the plane, what is the greatest number of triangles we obtain after triangulating them? In the figure below, there are shown all possible paths starting from the square and leading to the school. This is 166; hence, |A| = 166. Zvezdelina Stankova, A Decade of the Berkeley Math Circle: The American Experience (MSRI Mathematical Circles Library) (v. 3, 4. That's why letting them know their class is there to provide help and guidance means they can focus on the problem itself and not whether they are worthy to solve it. And all the variations in between and then some. And while having collaboration in the classroom sounds like a great idea, it doesn't just happen. Similar triangles. A student who processes information through movement, meaning that they have to fidget or move around the classroom to think, may benefit from mental math challenges that don't require paper, so they can get the wiggles out while learning in the process. The square removed from the lower-right corner of the rectangle does not change the perimeter of the polygon, but when each of the other five squares is removed, the perimeter is increased by 4. Do you think the answer is correct, and if so, how could you prove it? Prove that 2Tn − Sn = n. The shortest path between two points on the plane is the segment connecting them (Figure 7.1). The latter is much harder than it seems. Parallel lines. Daniel H. If a player is about to say a multiple of seven (like “fourteen”) or a number with seven as one of its digits (like “seventeen”), they must say “Buzz!” instead of the numbers. Indeed, there was such a film on YouTube (far from soap opera, though as she noted [My high school years were instrumental in my developing as a mathematician. First the students will need to understand the grid. This means that (2400 + 1)505 has the same last three digits as 1 + 505 × 2400 = 1 212 001. The best way to guide them on the climb so they can master that tree is by creating an environment where questions are welcome. TALENT As with all things, talent matters. It would be like saying you don't like reading when you've only read one style of book. There are five contests per year that run every month from November through March and can be administered and graded by the teacher during class time. I'm never sure that the process is effective. Proceeding with Heron's “shortest distance” problem (Figure 7.9), then introducing two parameters (the angles between the path segments and the sides of the square) and using some trigonometry, we get a function of one parameter. We asked a number of our AwesomeMath instructors and colleagues to choose from the following list of questions and share some of their experiences with you, the reader, because as we like to say, “When you engage in the trade of ideas, everyone improves.” Note that many instructors are currently college professors, so you will have a glimpse into the environments your students will be entering, and what better way to prepare those who are going to college than to learn from the problem-based approach of college professors! See what inspired them to enter the field of mathematics and share their passion for the topic, learn their approaches to PBL, and why they value collaboration. In the second bullet point in the example, Questions is where you take the time to make connections and relate what is learned to what you already know as well as make connections to other areas. That means letting them see you struggle and question in a positive way. Solution We have 72020 = (74)505 = (2400 + 1)505. It is worth mentioning that while the solution for up to four points could be done by ruler and compass, such a construction is not possible for more points in general position (a result obtained by applying Galois theory). 1. Since the problems are scalable and process is the focus over grades, they can work toward mastery instead of other false metrics. Chip Conley, “Anxiety = Uncertainty × Powerlessness,” Bit Think (May 22, 2013), Continuing, we find that 74n ends in 01 for all integers n greater than 1. Dr. Andreescu instilled the importance of community in his Olympiad training and continued with this approach when launching the AwesomeMath programs. Given a sequence of pairwise distinct real numbers a_1, a_2, ..., a_m, if m > n^2, then there is a strictly monotone sub-sequence of length n + 1. This rigor is easier to manage when you have a collaborative team (the teacher and fellow students) offering support and encouragement along the way. Hence, the sequence an = 1/2n, n = 0, 1, 2, ... is a possible match. One guard only tells the truth and the other guard only tells lies, but you don't know which is which. Library of Congress Cataloging-in-Publication Data is Available: 9781119575733 (paperback) 9781119575719 (ePDF) 9781119575702 (ePub) Cover image: Wiley Cover design: © Nadzeya_Dzivakova/iStock.com FIRST EDITION To our awesome community of colleagues, family, and friends who inspire us daily and made this publication possible. Finally, D2 = a2 + b2 + c2 = 22 + 32 + 62 = 4 + 9 + 36 = 49 = 72; hence D = 7. Also, 4! + 1 = 52 and 5! + 1 = 121 = 112, so the statement is true for n = 4,5. Remark. What is the greatest number of knights we can place on a regular 8 × 8 chess board without any two attacking each other? The mission needs to capture these goals so the stakeholders unite and connect to achieve them, heart and mind, and feel they can make a significant contribution. A, A, A, A, C, C, C, D, F, G, H, I, I, I, I, I, ... SOLUTIONS 1. This is the same type of flexibility required when solving a problem, so it only makes sense that is should be a part of the overall curriculum design. The first trick relies on the identity: n2 = (n − k)(n + k) + k2 We choose k conveniently, to get to the closest multiple of 10 or 100. Then x + (x + 1) + ... + (x + 6) = 130. For additional practice: Go to the website crorres/Archimedes/Stomachion/Pick.html This is an interactive geo board to reinforce this concept. Loading PreviewSorry, preview is currently unavailable. Then, γ = α + β. There are some interesting facts in Pascal's triangle: 1. Hence, consider the following triangulation. Since S24 = 576 and S5 = 25, 11S24 + 2S5 = 11(576) + 2(25) = 6336 + 50 = 6386. H., 6 Heron's formula, 219 Hero's Journey (Campbell), 38–39 Herzig, Emily, 6, 92, 124–125 Hexagonal numbers, 208 High School Mathematical Contest in Modeling (HiMCM), 105 Hook problems, 61 Hooke, Robert, 55–56 The Housekeeper and the Professor (Ogawa), 15 Hubris, 29 Human-centric pedagogy, 19, 36 Humility, 109 I Idealism, 76 Identities: algebraic, 87–88; capital in, 111, 113–114; Catalan's, 88; Lagrange's, 88 Imagination, 30 IMO. Squares are a common geometric shape for students to start with, and multiplying x*x is a first glimpse into exponents and more areas to investigate. The only known Wilson primes are 5, 13, and 563. Offer an application of these formulas into to a real-life problem. It relies heavily on range, rigor, and resilience to encourage curiosity, critical thinking, and creativity. www.mandelbrot.org MathWorks Math Modeling (M3) Challenge. Out of this number, only 4 • 3 • 2 • 1 words consist of letters M, E, O, and W. Example using the chart below: N Tn Sn Pn 1 2 3 4 5 6 1 3 6 10 15 21 1 4 9 16 25 36 1 5 12 22 35 51 So if we pick the third number, it would be as follows: 2(Tn + Pn) = 2(6 + 12) = 2(18) = 36 (or 62, the sixth square number). The first time the doorbell rang, only one guest arrived. Can we work backwards? Initially, creating the lectures can be a lot of work for the teacher, who may or may not teach the same classes year after year. Hence E = 7, the remaining digits are 2 and 5, and we can assign M = 5 and F = 2. Furthermore, when the number of spots is divided by the number of legs, the remainder is 3. Is there only one way to prove the Pythagorean theorem? Instead of worksheets with short exercises, timed tests, and/or reducing mathematics to arithmetic, students need to be exposed to meaningful and challenging problems where their personal strengths can shine. So how does this fit with autonomy? See Mini-units Lesson-specific questions, 97–98 Lines, 177–183 Listening, 108–109 Local peers, 120 Logic problems, 7; engagement with, 38–39; inspiration from, 8; relevance in, 19; in suggestion box, 18 Lucas, Édouard, 160 Luck, 55–56 M M3 Challenge. Life is not a zero-sum game (in game theory, this is the mathematical representation that one player's gain is equivalent to another player's loss). The next day, they can share their solutions and approaches with the class. 34 = 9 × 100 + (−16)2 = 1156 482 = 23× 100 + (−2)2 = 2304 752 = 50 × 100 + 252 = 5625 Solve using the second method: 392 572 632 782 Solutions 392 = 14 × 100 + (−11)2 = 1521 572 = 32 × 100 + 72 = 3249 632 = 38 × 100 + 132 = 3969 782 = 53 × 100 + 282 = 6084 The Number of Elements of a Finite Set OVERVIEW We often need to count the number of terms in a succession. This way, they themselves become active parts in the process of discovery. Further, when a lecture is recorded, students can slow it down, watch it multiple times, and/or skim over areas they know well. StudentWhen you are at the center of your own learning path, centric a student will have more accountability in the process and take more ownership of the outcomes. Why? You will see some answers that were included in other sections of the book; however, we are reprinting them here so you have all the experiences in one place. How many different values are possible for k? Kids progress at different rates, and since we aren't training racehorses but instead human beings, a problembased curriculum can scale to those different rates and students can begin to see their own strengths and contributions. Area and Volume of a Sphere OVERVIEW The area of a sphere of radius r is A = 4πr2. Parents would love nothing more than to have students who are curious go-getters who can support themselves and be able to think, and yet, we all know the stress and pressure put on kids to check that box of GPA and standardized tests or exam boards. Recordings are the most common way to create the lectures; however, not all students may have the same access to the technology needed to view them. Fagnano solved the case of four points. If T is a triangular number, then 8T + 1 is a perfect square: a. Take a look at this table: 27 6 95 1 438 Notice that if we add the numbers in each row, column, and diagonal, we get the same number. The top students from each Chapter Competition advance to the State Competition, which takes place in March. Thus, to show that 4 is nice, we need to show that 1 is nice, which is obvious. Unvarying; constant. Have students collaborate in small groups. 1, 3, 5, 7, 9 c. Just as it's easier to jog a mile if you've been training for harder races, when you learn the critical thinking that comes along with problem solving, then every other topic that requires math (and most do, in one way or another, e.g., economics) will be easier to understand. Making sure that parents are on board from the beginning will give them a stake in the process, but that's not enough. Added challenge! What are the last three digits of 72020? It also led our math team together to solve problems that our very dedicated math sponsors weren't sure how to solve in an efficient manner. It's not about always having the answer; it's about being a part of a team that is working together to understand the heart of a problem – it's a quest for truth. Following is a sample table answered by a student. Note that: Hence, The equation is reduced to (3n)/(3n + 1) = 21/22; therefore, n = 7. It's not about being inauthentic; in fact, it's about being more authentic to who they truly are. AwesomeMathacademy.org AwesomeMath Year-Round is a correspondence-based program that provides students with further opportunities to broaden their mathematical horizons, particularly in those fields from where Olympiad problems are drawn. The sequence an = 2n + 1, n = 0, 1, 2, ... is a possible match. That is because the things you know and the things you can control are in your power to change. Solution 1. Because r ≤ 11, we find that r = 5. 1, −2, 3, −4, 5 Solution a. Keep in mind, however, that learning is messy and each student climbs the tree of knowledge in a different way. Aspire to Inspire: Stories from Awesome Educators Just like collaborative problem solving can help students retain more information, think critically, and inspire them to be better, creating these learning environments can help teachers as well. Therefore, the last digit of the whole sum 1! + 2! + • • • + 100! is also 3. Some schools, states, and countries have required testing for all students where certain scores must be met to receive funding or other benefits, shifting the teaching incentive away from challenging mathematics and toward teaching to the test. That was the time period when my math abilities were recognized. The collaboration nurtures interpersonal skills, leadership traits, and conflict resolution, while providing interesting problems gives students a common goal to work toward where they can connect and share ideas. As an educator, there are always ideals to be reached when designing lessons, but they aren't called ideals for nothing – and many times we fall short of them. Second, the above proof shows that any polygon can be transformed into a square. Athletic coaches can split their teams into groups to work on different training exercises, and so can mathematics teachers. We have one main diagonal of eight white cells and six more white diagonals parallel to it. In language classes, students are able to peer review each other's work. HM ≤ GM ≤ AM ≤ QM 3. QUALITIES OF A GOOD LEADER Both the teacher and the students will be in leadership roles in a PBL classroom. We can write the Pythagorean theorem for both situations: The second equation can be rewritten as When we combine this with the first equation, we get 8. Theorem of Thales: If two parallel lines intersect the legs of an angle so that the resulting segments on one leg are equal, then the segments on the other leg are equal as well. Area Formulas Parallelograms 1. The letters are the first letters of the numbers (one, two, three, and so on). But we remember that if we add 1 + 2 + 3 + • • • + 100 (according to simple sums) we get 5050, which is greater than 7! 2. Twenty-one boys have a total of $200 in notes. The synergy that happens when students share with each other is a key component of an active learning environment. High School Mathematical Contest in Modeling (HiMCM). Consider a regular polygon such that the given polygon lies in its interior and use the idea in the previous problem. – then they start thinking about their interactions and reactions to new things in a different way. How can I look at this in a different way? Solution The numbers are 11, 12, 13, 14. Thus, we must find the last digit of 30!/107. IMPORTANCE OF TRADE Sharing mathematical insights is a type of trade, and when students engage in the trade of ideas, everyone learns more deeply and faster than they would on their own. How many times have you heard, “Will this be on the test?” or all the variations thereof, such as “Can you tell me what will be on the test?” or “Do I really need this?” The shift has focused from intellectual query and risk to playing it safe. Definitions Reciprocals. PARENT GAINS Critical thinking Multistep problems that can be solved in more than one way train students to think critically in order to work them out. Also, it helps students take ownership of their own learning, which not only helps in my class but in their college career in general. Not wanting to make mistakes. www.mathkangaroo.us Mandelbrot Competition. In Section II, “Teaching Problem Solving,” we will go into greater depth about the various math competitions and resources available. In particular, because the ancient Egyptians used them in such a way, the reciprocals of positive integers are referred to as Egyptian fractions. Discrete math, along with finite mathematics and linear algebra, are necessary to work in the modern world of computing. Parents want a “map of engagement” for how they can help their kids succeed with their mathematics homework. A characterization of right triangles: If a median of a triangle is equal to half of the side it connects to a vertex, then the triangle is right. Let's say that you've accomplished the first step of creating a welcoming collaborative atmosphere. Prove Viviani's theorem for equilateral polygons. After we have at least one solution for a problem, I like to ask the class what they think was the most important idea used in the proof. PEOPLE Some People by Rachel Field Isn't it strange some people make You feel so tired inside, Your thoughts begin to shrivel up Like leaves all brown and dried! But when you're with some other ones, It's stranger still to find Your thoughts as thick as fireflies All shiny in your mind! How Have Your Teaching Methods Evolved Over Time, and Why? The value of T determines E, the value of E determines I, the value of L determines M, and the values of O and R can then be determined to form a correct addition problem. It may take a while to get over these fears, making a flipped classroom approach take longer to show results. Using the correct values for I and b, the ratio n = I/b is an integer. The units provide an opportunity to: Ease into a PBL environment with manageable content. My father was always there with a more challenging puzzle. If the ratio of corresponding sides of two triangles does not depend on the sides chosen, then the triangles are similar. See also Teacher inspiration Integers, 83–84, 185–189, 199; co-prime, 227–229; factorials and, 194; nonnegative integer exponents, 88; positive, 255–258; properties of, 213; sequences and, 245–246 Integrity: for learning, 21–22; principles and, 110–111 Intellectual risk, 39 Interaction, 91 International Mathematics Olympiad (IMO), 3, 80, 106 International Olympiad in Informatics (IOI), 107 International Physics Olympiad (IPhO), 3 Interpersonal skills, 59, 62, 68 Intervals, 227 Introduction to Arithmetic (Plutarchus), 201, 204 Introspection, 30–32 Intuition, 163 Invariance, 168 IOI. More lesson-specific questions can be asked as well, and in Chapter 5, we provided prompts to help you when working with students. Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. Figure 7.9 Heron's “shortest distance” problem. While presenting the examples, I'm trying my best to show them how to think. It turns out that we need to carry the tens place over to the number on its left. What sparked their interest? Cauchy-Schwarz Inequality: with equality if and only if 4. Vincenzo Viviani (April 5, 1622–September 22, 1703) was an Italian mathematician and scientist. Remember, just because someone can play music doesn't mean they can instantly compose music; however, if you make the time to create, you will have a richer understanding and can make larger leaps in ability. 8. For example, do they exude more confidence or humility? “The Number of Elements in a Finite Set” miniunit includes many problems that require a lot of work and arithmetic. Example The figure below was made by gluing together 5 nonoverlapping congruent squares. A parallelogram is divided into two equal triangles by each of its diagonals. The mission statement can help set them on a clear path to achieve the requisite goals. A polygonal number is a number represented as dots arranged in the shape of a polygon. Collaborative problem solving will allow kids to blossom into leaders, adept problem solvers, innovative thinkers, process managers, knowledge facilitators, and more! The main point is that they have the autonomy to grow authentically for who they are. If you have students who receive quite a bit of supplemental enrichment in problem solving, their skills may surpass the experience of the teacher. Since T5 = 15 and P5 = 35, 2T5 + 2P5 = 2(15) + 2(35) = 30 + 70 = 100, which is the 10th square number. Problems Solve the following 1: Easy 1. I might ask them to think how they would prove the easier problem “A implies B.” After they manage that, I put forward the challenge of proving that statement C implies statement D. Now, I think that math is not only problem solving, but also the art of asking math problems. Confidence. This high school level test gives students one month to solve problems, and there are three rounds to the competition. Let the leftmost entry of the first row be y. S, M, H, D, E, H, I, K, M, ... 17. (2008). Have a Plan Goals should be few and strategies should be abundant. In mathematics, the art of proposing a question must be held of higher value than solving it. Using the table below to prove the example: N Tn Sn Pn HXn HPn 1 2 3 4 5 6 HPn − HXn = Tn–1 1 3 6 10 15 21 1 4 9 16 25 36 1 1 5 6 12 15 22 28 35 45 51 66 1 7 18 34 55 81 81 − 66 = 15 You can do the same for any number in the list. Collaborating with like-minded peers gives teachers much-needed connection, and, as mentioned previously, when you engage in the trade of ideas, everyone improves. A closed plane figure formed from line segments that meet only at their end points. Further, flipping the classroom will allow the collaboration necessary so that students can see their own mistakes and have feedback loops in place with you as the mentor and their peers as their helpers. Adding the remainder 6 gives 15 as well. See North American Computational Linguistics Olympiad New math, 68 Newberry, Anthony, 122–124 Newton, Isaac, 55–56 Nice numbers, 255–258 Nonnegative integer exponents, 88 North American Computational Linguistics Olympiad (NACLO), 106 Notations, 157 Note taking, 99–101 Numbers: ancient systems for, 9; consecutive, 185–189, 227; even, 185; Fibonacci, 18, 247; hexagonal, 208; large, 191; nice, 255–258; odd, 185, 218; polygonal, 205–211; positive real, 88; prime, 82–83, 201, 227, 253; Sierpiński, 253; square, 206, 209; squaring, 155–156; tetrahedral, 203, 249; theory for, 245, 253; triangular, 199–204, 206– 210, 246 Nurturing, of talent, 99 O Observation, 149 Octagons, 241–242 Odd numbers, 185, 218 Ogawa, Yoko, 15 Online peers, 120 Outcome-based learning, 2 Outcomes, 70 Outliers (Gladwell), 53 P Pacing, 37, 60 Pains: grades as, 70–71; from learning, 57; mini-units for, 61; for parents, 68–69; for students, 63–64; for teachers, 60–61. “In a growth mindset, people believe that their most basic abilities can be developed through dedication and hard work—brains and talent are just the starting point. There are some nice properties for parallel lines. Our problem-solving sessions in class were very strong. You're Reading a Free Preview Pages 61 to 65 are not shown in this preview. Students will jump to the worst conclusion when things are scary. GPA is a leading factor for college admissions, and so parents are less willing to give problem solving the time and energy necessary, especially if they feel that grades may be affected. But I was working a lot on my own! These three years in the school were successful for me from a competition standpoint as well, since I was selected to be on the Bulgarian IMO team in 1981. We don't want to train students for jobs that a computer can do better. Then, answer the following questions: 1. In this unit, we prove the famous Pythagorean theorem in several ways. Grace under pressure. All rights reserved. He is a professor of mathematics at Harvey Mudd College and is known for mental math capabilities and “Mathemagics” performances in front of live audiences. As a result, Pat's calculation of the area was too small by 35. Students, when the expectations are raised within a supportive environment of kind peers, will live up to their better selves and their peer relationships will strengthen. If students feel judged, ranked, or belittled in any way, they (of course) will be less willing to share and have the vulnerability necessary to take intellectual risks. Note that the triangular numbers can be justified by the following diagram: A triangular number is the sum of the first n positive integers: The nth triangular number is the number of distinct pairs to be selected from n + 1 objects. My favorite example for the

community for guidance and motivation. The advent of zero allowed algebra to be possible in the ninth century and physics in the seventeenth century. Five Common Mistakes Made by Every Problem Solver 1. When educating young math students, you can let them know that they are the captains of their ship, but as their navigator, you can guide them to really interesting destinations and expose them to a wider range of mathematics. The strategy to You don't solve the problem. Invariant. I am convinced that such problems are essential since everyone (even a professional mathematician) makes errors on an everyday basis. Consecutive. And, as you may have guessed, this won't require complete conformity (e.g., “do exactly as I say”) to a program. Since the current school curriculum delivers a narrow path of mathematics knowledge, climbing aboard the math competition train will expose students to a greater array of topics, including discrete mathematics, an area that incorporates both number theory and combinatorics (counting and probability). Not employing feedback loops 5. The following visual proof of the Pythagorean theorem shows that one can cut any two squares into finitely many pieces and reassemble these pieces to get a square. Thanks to these competitions, I met many life-long friends (my fellow students and my future college professors) and visited wonderful places in former Yugoslavia: Postojna cave and Portoroz in Slovenia, Sarajevo in Bosnia, Decani, with its famous fourteenth century monastery, and the Danube's Djerdap Gorge in Serbia. CE 46–c. Five congruent equilateral triangles could be positioned in a way to cover an equilateral triangle A (these triangles can overlap or run outside the triangle A). It is much easier to synthesize knowledge if students are given more time to process the new information. Since S4 = 16 and T12 = 78, 2S4 + T12 = 2(16) + 78 = 32 + 78 = 110. Every hexagonal number is a triangular number, but not every triangular number is a hexagonal number. Finding peers and a math community makes a huge difference in the lives of young problem solvers. C, D, I, L, M, V, ... 19. Find the area of the largest square. This will also help when developing your professional learning community (PLC). Figure 7.1 The shortest path between two points on the plane is the segment connecting them. That is: Thus, we have only two possible ways: Have students write algebraic representations of consecutive numbers based on the previous numbers. Here are a few types of kids and their interests to get you thinking: 1. The other two sides are called legs of the trapezoid. We write A and B as follows: In order to calculate |A| we need to find the largest k such that 3k ≤ 500. The midline theorem for triangles: The midline of a triangle is parallel to the third side of the triangle, and has half of its length. b. Instead of working through prescribed steps to solve exercises, problems require exploration – you need to be in a problem to solve it. This problem is a great exercise in thinking while also opening the door to more difficult forms of combinatorics (counting) problems. The symbol is !. Find the perimeter of the figure. Whether they are anonymous or during the problem-solving session, teachers will have the opportunity to keep their finger on the pulse of the students' abilities and know when they may need extra time or support material to help with certain types of problems. Students can also creatively produce a lot of things that don't necessarily have value. The most prestigious mathematics competition in the world is the International Mathematical Olympiad, which serves as a first-rate example of how community inspires excellence. Many times, another student can explain concepts more clearly since they have more empathy with what it takes to learn the concept, having just learned it themselves, as well as being more relatable since they are closer in age to the student they are helping. Further, it is inevitable that you will create some problems that you are unable to solve. Władysław Hugo Dionizy Steinhaus was a Polish mathematician and educator. If we calculate 7!, we get 7! + 1 = 5040 + 1 = 5041 = 712, so the statement is true in this case. 27 = 2 × 9 + 9; thus, 27 is nice. But 25 = 5 × 5, so the number of fives in 25! is 6. Innovation and creativity. EGO When engaging in a team atmosphere approach to math education, it's paramount to check your ego at the door. Figure 7.6 The system of paths can be reduced to the diagonals of the square... If the student was stressed when they were younger, what would be suggested as something they could do to take their minds off it? Students can learn about Pythagorean triples and their applications; properties and relations of integers; and the comprehension of three-dimensional geometry. Set by the district may make it difficult for teachers to have the necessary time to effectively start a flipped classroom approach. Know Your Approach LEARN FROM THE SYSTEMS THAT ARE SUCCESSFUL TODAY A number of widely successful and talked-about systems, mindsets, processes, philosophies, approaches (it's difficult to come up with one name or classification) are making a large impact in education, business, software development, and so on. Figure 7.6 The system of paths can be reduced to the diagonals of the square. The rest of the numbers are obtained by addition 2 = 1 + 1 = II 3 = 1 + 1 + 1 = III 8 = 5 + 3 = VIII 14 = 10 + 4 = XIV 20 = 10 + 10 = XX Solve 76 = 50 + 20 + 6 = Repeating a numeral up to three times represents addition of the number. The program is there to help kids from around the globe improve their problem-solving skills, so from the outset, we ask them to think about their goals in the program and what version of themselves they would like to be. Teaching through challenging problems requires rigor on both the part of the educator and the student, but also requires maintaining a balance between providing these problems and learning foundational concepts. She was given the resources to get started, and then she knocked this assignment out of the ballpark. Fear of failure cannot be an option because discovery is all about the honest testing of ideas and finding a path, pattern, or process. Have the students familiarize themselves with a chess board. Since you have a 12-sided figure, you lose the vertex and the two vertices adjacent to it. AMC 8, which is for students in 8th grade and below, and held in November. This kit includes: Classroom environment Knowledge bank CLASSROOM ENVIRONMENT Running a PBL classroom takes experience, so the habitat the students occupy needs to be designed and this section will help teachers new to this approach shorten their learning curve. How are things connected and how can I use those connections to find the answers? Try and understand the origin of their feelings so that they can move from irrational feelings to a more rational approach. Combinations Recall the combinatorics formula n choose k = n!/k!(n − k)! whenever k < = n, and which is zero when k > n. Then, a! + b! + c! > c!. You can't control the reactions of others or what they think. For example, one team member had a particular problem-solving skill set, namely, he excelled at number theory problems, so problems of this type were directed to him so as to work toward his strengths. What has changed? Neither the publisher nor author shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages. Dr. Emily Herzig, Instructor of Mathematics at Texas Christian University What Is Your Personal Approach to Teaching Problem Solving? Everyone knows a “dinosaur” kid, one who loves to study and immerse themselves with all the facts about dinosaurs, or the baseball kid who memorizes all the stats of players, and so on. I think as teachers we tend to forget how we learned. For example, great leaders inspire others to be their best, and that happens through high expectations infused in an environment of kindness. Math fear can be transmitted from wellintentioned parents, friends, or teachers, as well as originate from other more personal areas such as the following: Perfectionism. Find ∠ABC. It is very important to give the right motivation for considering the suggested problems. A man's errors are his portals of discovery. In similar way, for |B| we need to find the largest integer l with 7l ≤ 500, and we get |B| = 71. We put a positive integer less than or equal to 10 inside each cell of the table, so that the numbers in friendly cells are relatively prime. How can I use them to measure exactly 10 minutes? A grid is a set of points in the plane that are a distance of 1 from each other. For beginning piano players, listening to a concert pianist perform can ignite curiosity and inspire them to practice more. At this point, collaboration is critical. When your own curiosity is peaked, the students catch your enthusiasm and want to chase down the thought with you. You can make sure you take care of your physical needs (sleep and exercise) so that you are mentally ready for the challenge. Utilize mathematical modeling to understand the strategies employed by rational game players (decision makers). Not solving slowly when thinking quickly 4. When all of those areas come together, then success will happen. When you are in a leadership role, it means being more connected with the process because you are now responsible for the team's effectiveness as well as your own. So 2k + 1 = p and m = 1 + k = 1 − (p − 1)/2. Each square has an area that is half the area of the square that surrounds it. The pairs of opposite angles of a parallelogram are equal. Could they solve the problem at hand on their own? Imposter syndrome. Prove the equiangular polygons theorem. For example, in one we read that the problem of finding the shortest network connecting cities can be traced back to a correspondence between Schumacher and Gauss in 1836. Figure 7.3 Continuous curves connect the cities that are the vertices of a square ABCD. Our AwesomeMath Enrichment programs are filled with students whose schools placed a ceiling on their mathematics education and they are seeking outside resources. I assess the students' problem-solving progress through the discussions we have along the way and by the success they have in doing other related problems. Two lines that intersect at a 90° angle. The total measure of the angles in the triangle is 180°, so the region inside the circles and inside the triangle equals the area of a semicircle with radius 6. Try to make connections between problems. We thus conclude that 10 is also nice. Let z be the center entry of the first row. Suppose that there are at most three students born at the same day of a week. Every aspect of their lives will be touched by mathematics, from personal finance to big data to just understanding statistics in news reports, and both their careers and personal lives will require some level of mathematical competence. See Mini-units Lectures: in curriculum, 38; sage on the stage approach for, 20; by speakers, 18 Lesson plans. All we have to do is squish the numbers in each row together. Sequences can take a variety of forms and reflect many topics in mathematics. You can help your students find their Venn. They make a diagnosis and treat. 4. The crux of a successful PBL approach is, of course, finding meaningful and interesting problems. International Mathematics Olympiad (IMO). Definitions Right angle. Since the remainders of a number when divided by 2018 are 0,1, ...,2017, then we distribute these remainders into the following 1010 sets: If we choose 1011 distinct positive integers, then by the pigeonhole principle, there are two distinct numbers a,b with the remainders belong to the same set, i.e. Ai. If these remainders are different, then a + b is divisible by 2018. By showing them too many examples and too many solutions in class, I was not giving them the opportunity to learn how to discover such a long sequence of steps. If the pairs of opposite sides of a quadrilateral are equal, then it is a parallelogram. This means the teacher needs to be knowledgeable and encouraging, while also allowing ample time for students to work (and struggle) with the concepts and problems. They then need to form their group with one of the four different roles per group. Just like other competitions, such as sports, there are benefits when working toward common goals in a teamlike atmosphere that provides all team members with purpose and connection. Throughout MOSP, full days of classes and extensive problem sets give students thorough preparation in several important areas of mathematics. You cannot put a price or cost/benefit number on: Knowing that they are not alone in their love of knowledge and curiosity. Time Some students require more time to process and absorb information, so the time-intensive nature of solving difficult problems can be an obstacle. An identical box holds a single chocolate having twice the diameter of the chocolates in the original box. First few terms are 12 − 2, 22 − 2, 32 − 2, 42 − 2, 52 − 2, and the next term in the sequence is 62 − 2 = 34. An important theme of this work is that all important techniques and ideas featured in the book appear more than once! Olympiad problems don’t “crack” immediately. Taking the product of elements in each row, define the sequence , Prove that: a. In mathematics, are you actively consuming interesting problems with the goals of: Testing your skill sets in math competitions? Making the problems pertinent to what is happening in their lives (e.g., holidays, social events, etc.) creates more connection and interest. Hence, at most n marked points are single on their horizontal lines. Our teacher figured out that she was only challenging us on about 5% of the questions she asked and determined that she'd like to do more than that. The Pythagorean theorem is a central topic in early geometry courses. These numbers are arranged in such a way that every field contains a different number and that the sums of the numbers in each row, column, and diagonal are equal. When false metrics are removed, then students can be given problems that may normally be outside their grade level and take the intellectual risks to attempt the problem. And here is a short series of snapshots Jenny took during the lecture. Parents Today's parents are wearing a number of different hats with work, raising kid(s), extracurricular activities, after-school programs, and balancing their own lives. USA Mathematical Talent Search (USAMTS). By the pigeonhole principle, we must throw two dice 12 times to ensure that we get the same score twice. See also Polygonal numbers; Pythagorean Theorem; specific triangles Trust, 107–108 Turcas, George Catalin, 132–134 U Understanding: checks for, 150; critical thinking and, 24; information and, 13, 23; mistakes, 97–99; in PBL, 113–114; systems for, 45–46 Unit fractions, 255–258 Upfront work, 37 US National Chemistry Olympiad (USNCO), 107 USA Junior Mathematics Olympiad (USAJMO), 106 USA Mathematical Talent Search (USAMTS), 106 USA Mathematics Olympiad (USAMO), 106, 117 USA Physics Olympiad, 107 USAJMO. But it has to be used cautiously! I always encourage collaboration during the math circle I run. The shaded region has a total area 7. See USA Mathematics Olympiad USAMTS. The most important goal is that students have the joy of solving problems and want to concentrate on mathematics. – Titu, Kathy, and Alina About the Authors Dr. Titu Andreescu has been coaching, teaching, and training students and teachers for most of his exemplary career. There is no registration fee; it's been “fun and free since 2003.” The contest is held online and is unique because it is team-based and international in scope. Do they employ a specific technique to solve it? The revolution of design thinking was that better is in the eye of the beholder – namely, the end user determines the value of the design. 9T1 + 1 = 9 × 1 + 1 = 10 = T4 9T1 + 1 = 9 × 1 + 1 = 10 = T4 b. It is not a collection of very difficult, and impenetrable questions. I use more inquiry-based lessons, where students will look at data or create a table of information and try to determine a relationship or pattern. Hence, n is a divisor of 360 that is greater than 2. It is of course incredibly useful to be able to gauge student learning on the spot by hearing from them what they did and did not understand. If the memo or notes feature on your phone or computer is your preferred choice, then by all means, use it. Following one another in uninterrupted succession or order Integers. We find that in each row of Pascal's triangle, n is the row number and k is the entry in that row, when counting from zero. 2. 10, 11, 12, 13. Find the value of n. See also specific topics MathWorks Math Modeling (M3) Challenge, 105 Meaningful problems, 75–79 Means, 227 Meditation, 94 Mental mathematics, 155–156 Methods: Pigeonhole principle as, 228; for teachers, 123–125, 127, 130, 134; trial-and-error, 163 Metrics, for learning, 48 Metroplex Math Circle, 112 Midlines, 90 Mindsets: fixed, 27–28; growth, 35; what if, 78 Mingus, Charles, 10 Mini-units: cryptarithmetic, 151–155; elements of a finite set, 157–159; equilateral polygons, 168–170; magic squares, 159–163; math and chess, 170–171; for pains, 61; for PBL, 147; Pick's theorem, 165–168; polygons, 168–170; questions for, 147–148; roman numeral problems, 148–150; spheres, 172–173; squaring numbers, 155–156; toothpicks math, 163–165 Mission statements, 41–43 Mistakes: in PBL, 23–24; phobia of, 97; understanding, 97–99 Modeling, 49, 93, 121 MOEMS. There are many online PLCs for PBL, active learning, or competition training. That doesn't mean it is a free-for-all! Fundamentals need to be covered in a given school year, but there are ways to maintain foundational learning to get them through grade-level requirements (short view) while maintaining their love and interest in the topic beyond what is expected in the school curriculum. We have to find |A∪B|. During World War II, a tribe in the South Seas witnessed cargo being dropped by airplanes onto islands where temporary air strips had been built. Solution A common problem in mathematics is to find a formula or general rule for constructing the terms of a sequence. A view of the box is shown: 1. The sequence an = (−1)n–1 × n, n = 1, 2, ... is a possible match. What can't you control about the fear? Algebraic proof of Problem 8: 9. 1). In the square A, there are k students starting to go to the school. Those who prize creativity want the full story and tend to think in three dimensions. However, if among the five numbers we only have two distinct remainders when dividing by 3, by the pigeonhole principle there must be three among them that are congruent modulo 3. They are 5, 10, 15, 20, and 25. This is all about searching for the truth in problem solving, which removes personalities from the equation and shifts the focus to solving the problem. Share your experiences. Deeper learning When made to think critically, they learn concepts at a deeper level than they would in a broadcast environment. Over the years, I became increasingly confident that inspiring students is more important than lecturing, as is letting the students present their solutions in front of their peers, instead of me presenting all problems. We consider all possible distribution of points in the plane. Help Your Students with This Five-Step Self-Assessment Process: 1. Do they understand how and why the formulas work? (Math Leads for Mathletes, Book 2, page 48) Providing a range in topics and connecting mathematics with other disciplines, such as history, will ignite students' curiosity to dig deeper and find the beauty and relevance of the mathematics they learn. The advice and strategies contained herein may not be suitable for your situation. These ideas are the tip of the iceberg. Answer: No, because it will need 63 moves to arrive at the opposite corner and at each move the knight changes the color of the square it occupies. To combat math phobia and “fight, flight, freeze,” you want to create an atmosphere where there is no fear of mistakes, only the quest for truth. I also don't want the students to think about what they're going to score with the time they should be spending thinking about problems. The following Think Tank is a counting problem (i.e., combinatorics, a topic in discrete mathematics) that uses the principle of inclusion– exclusion, and if the kids use logic, they can figure it out. They will learn that consecutive forms include even consecutives, odd consecutives, and multiple consecutives, and they might come up with interesting conclusions. Again, when the curriculum is about the information and not about judging the student personally, then there is less posturing and concern over place. Prove that it is possible to find two boys who have the same amount of money. As mentioned in previous sections, this is short-sighted thinking when raising life-long learners. Parents can be a part of the process. Straight angle. The number of elements of A, in our case n, is called the cardinal number of A. If N = 5, then we have a carry of one out of the ones column, and the tens column gives T + 2E + 1 = T, T + 10, or T + 20, all of which are impossible. Thus, ∠BAC = 80°. Now imagine a circle that is divided in four equal parts by two lines AB and CD that pass through P. Another term used in education for average is standard. Years ago, I was enthusiastic in doing problems where the solution was clever, unexpected, and beautiful. In this role, she has had various speaking engagements as well as managed multiple communication channels online, where she discusses mathematics education with parents, teachers, students, and businesses. In particular, I listen to all ideas proposed by the students during the discussion and draw their attention to the ones that are interesting and useful. They can be the leader, idea facilitator/questioner, innovator, or thinker, etc. When you look at the types of enrichment centers, they serve the following purposes: Tutoring centers help struggling students get up to speed. Hence, F and S being consecutive must be 2 and 3, or 3 and 4. Provide question feedback. Only a few participants are capable of solving all the problems; yet, through the attempt everyone learns about the real world of mathematical research. When you are at this level of ability, however, how do you make this community come together? These strategies require diplomacy, and a good leader knows how to achieve consensus so that the team works together effectively and patiently to solve the problem at hand. As adults, our day is split in spending time in the following areas: Academic capital. These problems help engage the class and get them thinking! I have two hourglass clocks filled with sand. What could my number be? Meaningful When the problems connect to real life for the students problems and they see the use beyond the class, they are more curious about the material. The angles at a base of an isosceles trapezoid are equal. We shall provide an example for n = 1011 as follows. Then it must be that C + 1 = U + 10, and it follows that U represents 0 and C represents 9. Area. Write 2 × n! = 2n × (n − 1)!, then 2. You're Reading a Free Preview Pages 191 to 198 are not shown in this preview. Because she didn't want them to rank each other by being at Level 1, 2, or 3, instead, they would be seated at tables labeled as famous composers and the rank was based on when that composer was born. The area of the big square in the figure is (a + b)2. Autonomy 2. Teachers needn't, however, shoulder the burden of managing the club alone. We only need to prove that numbers A, B, and C satisfy the Pythagorean theorem. The square of the nth triangular number equals the sum of the cubes of the numbers 1 through n. Challenging mathematics in an environment that is kind, collaborative, and respects each student. The six students representing the United States of America in the IMO are selected on the basis of their USAMO scores and further testing that takes place during MOSP. Players take turns counting up from 1: the first player says “One,” the second player says “Two,” and so on. reflections/ Purple Comet! Math Meet has been “fun and free since 2003.” This annual, international, online, team mathematics competition is designed for middle and high school students. If the question calls for consecutive even numbers, we would have to “make sure” that the number we choose is even. When running a mathematics club, a student asked this exact question, even though her mother had majored in math when she was in college. DON'T FORGET GRACE When you first start writing problems, you may not always like the outcome, and you'll be even more critical of early problems you write as your skills improve. It shows students how to look for a good start when solving a problem. From the inclusion-exclusion principle, it follows that |A∪B| = |A| + |B| − |A∩B| = 15 + 16 − 10 = 21; hence, the number of students in class is 21. The “Pigeonhole Principle” unit requires students to write mathematical proofs, not numerical answers. Definitions Consecutive. Various employment combinations such as both parents working, single income, part-time, work from home, etc. Hence ∠DEX = ∠D'E'X (where the point X lies on the extension of EE′). Let's define a set P as following: a. Thus, ∠A + ∠B + ∠C + ∠D = 360°, hence 70 + 80 + 90 + ∠D = 360°, yielding ∠D = 120°. They could, however, learn to be physically fit and embark on some gymnastic moves that fit their body and their interests. See also Game theory Think tank: counting problems in, 20; PBL in, 39; scaffolding in, 9 Three C's. Therefore, This implies that 20N + 20 − 2 × 251 = 210 − 252. Summary We studied some facts about Pascal's triangle: triangular and tetrahedral numbers, Fibonacci sequence in Pascal's triangle, powers of 11 in a Pascal's triangle, powers of two and sum of entries in any row of Pascal's triangle, considering Pascal's triangle modulo 2, and combinatorial aspects of Pascal's triangle. So while you may like components of the mission listed, your own class, school, district, and personal style will shape the mission to be most relevant to you and your students. An ordered list of objects. It's also good for kids to see you struggle, evaluate a new approach, and enjoy the journey with them. www.arml.com There are many more mathematics competitions at the local levels and, of course, others that are not listed. FOUNDATIONS Cryptarithmetic Conventions 1. Properties and characterization of rectangles: The diagonals of a rectangle are equal. Todd Rose, director of the Mind, Brain, and Education program at the Harvard Graduate School of Education, where he leads the Laboratory for the Science of Individuality and author of The End of Average: How We Succeed in a World That Values Sameness. In 1202, Pisano showed that every rational number can be represented as a sum of distinct Egyptian fractions. When did all four ships meet again at the port? Sequences exist throughout all academic areas. In the past, I enjoyed helping students figure out really hard mathematics problems and understanding the beauty of the science. Even as adults, we love to play and compete and solve problems with friends. He was known for contributions to set theory, number theory, theory of functions, and topology. One of the most famous sequences is the Fibonacci sequence which was created by Fibonacci in order to explain how fast rabbits could breed under certain ideal conditions. So k = p and m = 1 + k − 2k2 = 1 + p − 2p2. You're Reading a Free Preview Pages 90 to 104 are not shown in this preview. Hence, d + a + b + c = 9. The beauty of this style is that it also accommodates nontraditional learners. Dr. Mirroslav Yotov, Assistant Professor of Mathematics, Florida International University What Were Your Own School Experiences Like in Your Country That Contributed to Your Love of Problem Solving? Later, I discovered that the math Olympiad exists with even more of these types of problems. If the diagonals of a trapezoid are equal, then it is isosceles. Continuous A problem-based curriculum provides challenge levels challenge for all the students in the class, and since the focus is on process and not outcome, problems can be harder. Purple Comet Math Meet! 2019 contest, . Problem solving prepares students for the world they will face, which is a continuously changing nonstandard landscape that requires dreamers, innovators, entrepreneurs, and in all areas, problem solvers. What led to their big discovery? SOLUTIONS 1. Between after-school programs, sports, enrichment activities, family obligations, and schoolwork, it is difficult for them to have the brain space to focus on hard problems in a significant way. And as was discussed in the “Pains” sections for both teachers and students, time, busy schedules, and the emphasis on standardized testing can make it difficult to effectively implement a problem-based curriculum. The USAPhO is an exam for US residents by invitation. When you discount the abilities you do have as just luck or a fluke and believe that you really aren't good at math, just situationally serendipitous. However, there is plenty of research available that shows how this approach is more conducive to creating tomorrow's thinkers than the rote or arithmetic-centric curriculums they may remember when they were in school. Retention Retention is much greater when working at a more challenging level and engaged in the resilience of problem solving. It means viewing your students each as a collection of strengths and not a collection of weaknesses. This is free contest. The remaining interior angles are all equal in size and have an integer value. And it is this process I'll try to share with you now. Those that have an unexpected answer and a clever explanation. They are usually held on college campuses and have a problem-solving focus. Let a, b, c, and d be the square of some consecutive positive integers. 927 498 = 9 − 2 + 7 − 4 + 9 − 8 = 11 is divisible by 11 12 – must be divisible by 3 and 4 Pigeonhole Principle If kn + 1 objects are distributed among n boxes, one of the boxes will contain at least k + 1 objects. It's particularly empowering when students don't have to follow a prescribed set of steps to solve a problem but can try different approaches and discover what works. Collaborative teams create a “back of the room” feeling where girls can work together and not be talked over, but instead, inspired, and willing to connect. For example, the sixth heptagonal number (81) minus the sixth hexagonal number (66) equals the fifth triangular number (15). At that point, human time was squandered so as to maximize the efficiency of the machine or factory. Parents and educators will read books beyond a student's personal reading level so that they can hear the richness of language and be exposed to more intricate sentence structure. Preparation and practice can counteract the dread and worry and grow confidence. Also, 16 = 2 × 7 + 2, but 7 is not nice, and this route does not descend to a nice number, so we go to the other route to prove it is nice. When you teach in front of the students, you usually feel, if you should select something easier or more difficult for the next time. At the age of 17, he was an assistant of Galileo Galilei. Solution What cave would the other guard tell me to take? It is very important to give them loads of tests and feedback on their performance in tests. Sometimes only a few terms of a sequence are given and the goal is to identify the sequence. In class, there are three main ways I help students practice their problemsolving skills: 1. All lessons are intended to engage and delight mathematics students of all ages. Solution 2: By tessellating the large square, we get a diagonal grid of squares in which 1 out of 8 is shaded. If you aren't enjoying the process, your students won't enjoy the process. Lagrange's Identity: 18. Let Find N. Be sure to tap your support network: Your students, colleagues, and friends can help beta test your ideas as well as provide the support to keep going! You don't want to get bogged down in perfection by writing the best problem. What is the probability that the keys Meow jumped on can spell its name? This is because PBL utilizes problems that are relevant and meaningful to the workforce our students will be entering as well as the workforce in which their parents currently operate. The latter equation reduces to (n − 7)(n + 6) = 0. Learning Objectives Prove the Pythagorean theorem and apply it in order to find the length of the solid diagonal of a rectangular box. How are creativity and constraints related? They need to be able to think and notice shifts in data and operations so that they can add value whether they work in technology, construction, agriculture, manufacturing, the service industry, or corporations. Example 2 Solution Looking at the tens digits, we must have O + W + 1 = O + 10, with carryover (O cannot be zero, otherwise W would be zero as well, a contradiction). If a math circle does not exist at your local university or college, again you can approach professors in the mathematics, statistics, computer science, and/or other STEM departments to see if they would be willing to start a circle. Now that we have introduced the new notation we will refer to a sequence as (an)n ≥ 1. There is a registration fee associated with these exams, and as well as a per bundle (10 tests) fee. Move two toothpicks in such a way as to obtain five congruent squares. None of the numbers shown contains the letter E when spelled out. If the question calls for consecutive numbers, you can let one of these numbers be x. Each chocolate is wrapped up in paper. The problem was interesting, and I knew the solution could be obtained by trying all 100 cases; however, I have always been “lazy,” so I remember sitting there thinking: there has to be a better way that gets me to recess sooner. That is in a sense more important than having the bright idea for answering the question; the right questions prompt interesting ideas! For the motivating portion, I usually start with introductory and explanatory (not necessarily trivial or easy) problems. Purple Comet! Math Meet contest 2019. This means instead of correcting work, you are asking the student and/or the class, “can this be true?” or “is there another path that gets us closer to truth (the solution)?” I know this may seem like splitting hairs by changing the wording, but the facilitator of truth is leading students to where you know they can be instead of telling them where to go – the distinction is a critical one to have a successful PBL environment. I was bored and would then be disruptive during the class, so the teacher gave me more problems to solve, although they weren't very difficult. From the following picture we know that x, (2 − x), and 1 form a right triangle. In this class, I assign a group project for which I give them almost the entire semester. Working in groups, students are more willing to explore, make mistakes, and help each other. In Eastern Europe, math circles have been using this problem-based approach for teaching mathematics for decades. 111, 112, 113, 114. Assessment, assessment, assessment. The diagram shows a large square with each of its sides divided into four equal segments. In other words, modulo 1000 is congruent to 505 × 2400 + 1 = 1 212 001 (binomial theorem), so the last three digits of 72020 are 001. Solution 9! + 11! = 9!(1 + (11 × 10)) = 9!(111). Thus, collaborative learning in the classroom could be key to closing the achievement gap and allowing capable but underprepared students to reach greater success in math. Nice numbers involve working with some tricky arithmetic, understanding a layered mathematical definition, and constructing elegant solutions. Adapted from Walter Pauk, How to Study in College (7th ed.) (New York: Houghton Mifflin Company, 2001), . Students who are strong in math and train hard to learn and grow should be able to have the same pride and be a part of an environment that can offer additional challenges, namely scalable curriculum. A triangle with side lengths 16, 18, and 21 has a circle with radius 6 centered at each vertex. No airplanes land. So an extra caveat must be added of active versus passive. Find |A∩B|. The problem requires the student to use multiple pieces of information from their knowledge base and combine them to calculate the complement of the area, which is a concept used frequently in mathematics. When your main concern is each student improving every day, you can evaluate how that progress looks for that particular kid. Top-scoring USAJMO students are also invited to train at MOP. Show that every perfect square is nice. As a framework, it will not be a comprehensive resource; rather, it will show you what areas are required so that gaps in knowledge can be discovered and filled. It's the same in education, and as an educator, you can see where each student falls on this spectrum and help guide them in their journey. That's why in the Hero's Journey, the hero will initially refuse the call to cross the threshold; they feel like they aren't the right person nor have the right skills. Solving for x (by inspection or the quadratic formula), we find no integer solutions. The second method used for fast squaring of numbers 30 through 80 is based on the following identity: n2 = 100 × (n + 25)2 + (n − 50)2. Set up a system where you call on students randomly. What was their childhood like? Thus, the sum of the angles is 4 × 180° = 720°. In some cases, working backward from the desired result is helpful. A ladder rests against a wall so that its top is 42 ft from the ground. STEM peers. Whether running the collaboration team (student) or running the class (teacher), it's important to understand the qualities of a good leader. Consider similar triangles obtained by drawing an altitude on the right triangles' hypotenuse as in the following figure. So how do you help students (and, in turn, how they can help themselves) get over this fear of math? Show that if n is a nice number, then so is 2n + 2. Teams must be comprised of three to five students and one teacher-coach with a maximum of two teams per high school. Authenticity: The Cargo Cult Science Trap Authentic learning means more than just going through the motions; you don't want your students to fall into the cargo cult science trap. It's not only important for teachers to move away from judging work as right or wrong, the students need to be encouraged to leave this fixed mindset at the door, but they also need to keep students focused on the journey and not the destination. All answers are non-negative integers. Determine the area of a triangle whose side lengths are 13, 14, and 15. How does this affect the interior angle measurement? The National Association of Math Circles offers a free resource guide, Circle in a Box,6 with tips for starting a circle and free session plans. Since X + Z = 9, that means Z = 8. So, let us divide all the set in the following subsets: (1,2,4,8), (3,6), (5,10), (7), (9). Dr. Loh, the current leader of the US IMO team and a professor at Carnegie Mellon University, decided to take this MOP invitation even further and invited teams from other countries to train with the US team. Primes of the form ((2)2)m + 1 are called Fermat primes. Learn to write math, as well as to talk about mathematics for different types of audiences. When I started working with mathematically gifted kids, I would spend approximately 20% of the time on inspirational stories, 0% of the time letting students present, and 80% of the time on lecturing. This will entail a lot of consumption of math problems to gain the inspiration to create your own, but the best way to get started is to take that first step! And with all first steps, having a supportive community can make all the difference. Having a strong bedrock of principles, which can be decided ahead of time when creating the core values document, allows each leader to work within an integrity framework agreed upon by all participants. From a/m = c/a, we find a2 = cm. Example 1 Prove that no matter how we pick six numbers from the set {1,2,... 10}, there will be two among them, say a and b, such that a + b = 11. 28 = 2 × 10 + 8, we showed before that 10 is nice; thus, 28 is also nice. Math circles began popping up in the United States in the 1990s, and there are now hundreds of math circles across the country. Having an “accountabili-buddy” will let students know they have support, and stronger students will learn information more deeply in these tutoring roles. Prove that a1 − a4 = a5 − a2 = a3 − a6. Anthony Newberry, 21 Years Teaching at Hirschi High School, Wichita Falls, TX What Were Your Own School Experiences Like in Your Country That Contributed to Your Love of Problem Solving? There is no generalized formula for factorials, and we almost always have to calculate them using multiplication. MISSION STATEMENT AND CORE VALUES Why would a teacher want a mission statement and set of core values created for their classroom? Many times, the roles are determined by the personal characteristics of each individual, but if possible, students should strive to wear different hats so that they can grow in different environments with varied expectations. Design thinking. I believe that opportunities for students to articulate ideas with their peers are vital to each student developing a personal understanding of the material. A sequence, such as the positive odd integers 1, 3, 5, 7, ..., in which each term after the first is formed by adding a constant to the preceding term. They imitated the actions that they believed made the cargo appear out of bamboo and local materials. Example 2 Calculate 2T10 − S10. Index A Academic capital, 113–114 Accessibility, 16 Accountability, 111 Active creativity, 54 Active engagement, 62, 68 Active learning, 59 Active learning environment, 91–97 Activities, 150 Additional practice, 166 ADHD. An angle measuring 90°. In this interactive learning environment, students will also be working together, so not all of the explanations need to be made by the instructor. Find the greatest n for which n! ends in exactly 33 zeros. The execution of the idea misses the mark. The reason is because students still work at their own pace. See Math Olympiad for Elementary and Middle School Monomyth (Campbell), 38–39 Monroe, Marilyn, 150 MOP. I didn't care too much about the context and the motivation. As you go through these questions with a student and when you recognize that a student has indeed made an error, especially ones involving reasoning, not a mere arithmetic mistake, it's important to not interrupt the student right away and instead give them the opportunity to correct themselves. From the product of two consecutive numbers if you subtract the sum of the same two numbers, the answer is 5. Do they look for patterns or start writing down ideas? Solution: 2019 = 1000 + 1000 + 10 + 9 = MMXIX 2. This is indeed true because 5. Also, 45 k ≤ 2009 gives k ≤ 44, and 63k ≤ 2009 gives k ≤ 31. Example 2 Prove that among any six numbers from the set {1,2, ...,10}, there exist two numbers that are coprime. The membership fee is per team (35 or fewer students consist of a team). Wilson prime. At the AwesomeMath Summer Program, when students reach various skill levels, they delight in solving Lemmas on their own (subsidiary or intermediate theorems used in an argument or proof) that already have published proofs to test their abilities and see if they can do it. If King Kong is four times bigger in each of the three dimensions (4 times taller, 4 times wider, and 4 times longer) than the average gorilla, that equals 400 multiplied by 4 multiplied by 4 multiplied by 4, i.e., 400 × (20/5)3 = 25 600 pounds. Three consecutive multiples of 5 have a sum of 75. Alternatively, let y be the fourth integer. Group exercises. From 9k ≤ 2009, we get k ≤ 223; that is, |C| = 223. What Are Your Favorite Problems, and Why? Do there exist two consecutive odd numbers with product 45? Find the distance between two opposite corners of a 2 × 3 × 6 rectangular box. (This can be proved by using the analysis concept of continuity and the geometric properties of plane figures.) Vlad Crisan, PhD Student in Mathematics at University of Goettingen in Germany, with Bachelor's and Master's Degrees in Mathematics from the University of Cambridge in the United Kingdom What Is Your Personal Approach to Teaching Problem Solving? As mentioned in Section I, a mission statement and core values will help in this regard; however, appealing to a student's personal long-term goals of networking and connection based on an investment of time in their identity capital will aid in producing a cohesive problem-solving group. The people you choose to surround yourself with have a lasting impact, so to be successful, it is critical to be a part of a community that values your contribution and, in turn, you value theirs. Since T3 = 6 and S5 = 25, 4T3 + S5 = 4(6) + 25 = 24 + 25 = 49. Telos is from the Greek and means purpose or goal. Prove that 2(Tn + Pn) is a perfect square. Unit fraction. Another way of looking at this is to use an equation: implying that Y + 9Z = 89X. How does this work mathematically? So we bring in these peers, who are actually at the same level as these top six. An important aspect of a problembased curriculum is having students present their work in front of the class. Figure 7.7 The first reduction of the system of paths should look like this ... Observe that 1/4 + 1/4 = 1/3 + 1/6 = 1/2, and then solve the following exercises. This problem is about basic measurements; however, many students get it wrong by rushing to provide an answer without thinking about what really is being asked. Students are learning critical life-long skills: Effective communications. There are so many incredible mathematicians in every community who are willing to donate their time and energy to share their love of the topic with young students. Other times, the school is concerned that if students move two or three levels ahead, by the time they are seniors they will have run out of classes to take. Just as every football player cannot be the quarterback, not every student is going to excel in the same way with mathematics competitions, but this brings us back to the focus being placed on the process and not the outcomes. Cope: Develop a plan for how to handle math phobia in the future. This is where having feedback channels is so important. Any of these three areas would provide access to like-minded colleagues. Foundations of Number Theory Divisibility Division Algorithm Primes The Fundamental Theorem of Arithmetic C.D. Euclidean Algorithm Bezout’s Identity C.M. The Number of Divisors The Sum of Divisors Modular Arithmetics Residue Classes Fermat’s Little Theorem and Euler’s Theorem Euler’s Totient Function Multiplicative Function Linear Diophantine Equations Numerical Systems Divisibility Criteria in the Decimal System Floor Function Legendre’s Function Fermat Numbers Mersenne Numbers Perfect Numbers2. When faced with this challenging problem: What is their approach? If you create an environment where the expectations are set ahead of time for kind, positive guidance and collaboration, then the result is a room full of firefly thoughts and not brown, dried leaves. Lucius Mestrius Plutarchus (c. Heroes are the protagonists in the story, which means they need the opportunity to be front and center. This is because they are no longer just memorizing concepts, they are developing them and so they are able to reconstruct what they've learned and/or derive formulas themselves, because they understand how they were created. When the lens changes from the person to the problem, students won't fixate on whether they are personally good or bad at math, and instead will seek out what is needed to rise to the challenge presented. How do you give them the compass to know if they are going too far off trail? How many times must I throw two dice in order to be sure I get the same score at least twice? What What can you don't you control? The following two problems were created by Dr. Andreescu to incorporate the year as the constraint (math competitions are particularly fond of these kinds of problems): 1) This problem uses the year 2019 as the constraint: Find all triples (p, q, r) of primes such that pqr − 18(p + q + r) = 2019. My earliest memories include some in which my father was reading the newspapers where Romanian students returning from the IMO were interviewed and he was speaking to me about them with great admiration. The important thing to stress is the journey of discovery, looking at what works and what doesn't, and having fun. As with all things, practice is the best way to shape skills. (Any number

this area is to add the areas of the four right triangles to the area of the square with side c. Thus, the equation R + 2T = X + 19 shows that R is odd, hence R = 7 and 2T = X + 12. Using symmetry reasoning, we can place at most seven bishops that do not attack each other on black squares. Classes over 20 students present a challenge to teaching a problem-based curriculum, especially if the teacher is inexperienced in classroom management. It can bridge learning to other areas of academics and life. The “Dissection Time” unit features problems that require very creative solutions. It's a specific process, but effective when you want to relate knowledge to other areas, as well as be able to apply what is learned in various ways. 1, 2, 4, 8, 16, 32, ... 3. Find the least of these numbers. Competitions such as MATHCOUNTS and AMC allow you to increase your community as you increase your skills. 9T1 + 1 = 9 × 1 + 1 = 10 = T4 Because T1 = 1 and T4 + 10 then 9(1) + 1 = 9 × 1 + 1 = 10 10 = 10 = 10 b. Only I, X, C, and M can be repeated; V, L, and D cannot be, and there is no need to do so. The answer is designed in the following figure. This book could convince you that math can be fun.” Raj Varadarajan, Senior Partner and Managing Director, Boston Consulting Group “This book is a brilliant road map that delights in its own theorem of authenticity and relevance. Therefore, 20N + 20 = 210 + 250, thus, N = (1024 + 250 − 20)/20 = 62.7. 5. Therefore, there is a maked point that lies neither single horizontally nor single vertically. Having problems that are scalable for all levels and/or able to be “chunked” into more manageable pieces as well as have multiple approaches to the solution will increase class discussion and connection. Thus, (k + 1)(2a + k) = 200. Below are problems that fit these categories. Thus, we have 7 holes. Dr. Feynman was an advocate for understanding the beauty and depth of problems, the underlying why, so that truth can be discovered. So if you want to calculate 6 choose 3, look at the 7th row, 4th entry (since we're counting from zero), and you'll find that the answer is 20. In mathematics, they would benefit from seeing visual proofs or deriving formulas or creating their own problems for the class to solve. By thinking about it, they internalize the fundamental concepts much better. Metroplex Math Circle is a free program that was designed to attract gifted students and educators in the Dallas/Fort Worth area to provide an avenue outside the standard curriculum to develop their mathematical and problem-solving skills. The AwesomeMath mission statement allows us to tell our customers, parents, and educators what types of students are a fit and what products/services we offer, and it sets the tone for the environment and community we like to create: AwesomeMath is devoted to providing enriching experiences in mathematics for intellectually curious learners. HPn − Pn = Tn–1 or Pn − Sn = Tn–1 Just replace the variables with the appropriate number. You can always start with exercises as a warm-up, but then ramp up the challenge to more thought-provoking problems. When teaching, I try my best to lecture as little as I possibly can by briefly introducing students to main concepts and then assigning problems and exercises that aim for the students to get their “hands dirty” and discover for themselves what those concepts really mean and how they fit in the mathematics field. 2, 5, 14, 41, ... 6. I think as teachers/professors we tend to forget how we learned. You can download the paper by clicking the button above. There was a study in 2011 conducted by the Royal Society for Medicine by Sir Ken Robinson where participants were asked a simple question, “What can you do with a paperclip?” Adults were only able to think of a dozen or so uses while kids were able to think of hundreds of possibilities. Praise students for asking all levels of questions, because if they are confused, chances are there is another student too shy to ask. Context while explaining mathematical concepts is also important, so it's not just range within the topic of mathematics but range outside the topic as well. A reciprocal, or multiplicative inverse, is simply two numbers that have a product of 1. Properties and characterizations of parallelograms 1. What would the next consecutive number be? 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The area of a rectangle is equal to the product of two of its adjacent sides. This strategy addresses many issues, such as: Having students answer the question: When will I need this? The United States of America Computing Olympiad (USACO) finalists are selected via a national competition and invited to a rigorous academic summer training camp to further improve their skills. Do they need more information to understand the problem? (Example: [1, 2] is an interval including both of its end points, but [1, 2) is an interval that doesn't include 2 as one of its end points; however, it includes the other end point, i.e., 1.) Integer. But then if we try to divide 103 by 7 we do not get an integer number. It is clear that each of the four triangles could be covered by one of the before-mentioned equilateral triangles. Solution We can draw one diagonal to dissect an (n + 1)-gon into an n-gon and a triangle. This sequence is of powers of two. Carol S. First, let's start with a personal question: What Teacher Made the Biggest Impact in Your Life, and Why? LEARNING OBJECTIVES By the end of the lesson, students will know basic operations with Roman numerals and be able to convert years from Roman numerals to Arabic numerals and the other way around. He'll try a different strategy or ask his friend for advice or go online and watch YouTube videos. Sometimes, I tell them they must fight with the problem for 10 minutes before I will give them assistance. Students must feel worthy at the outset, and this happens when they are in a supportive environment with positive guidance. 1, 3, 7, 15, 31, 63, ... 5. The problems themselves should be challenging, interesting, and yet doable, while also illustrating key ideas of the material that needs to be covered. Solution You know the height difference (one dimension), but you need to translate that to the difference in volume (three dimensions) of the gorillas. Sequences are used every day in fields such as biology and computer science. Resources could be to create a flipped classroom video of the lecture to be viewed at home and the problems can then be the focus of the class. Solution We recall that the volume and area of a sphere of radius r are V = (4/3)πr3 and A = 4πr2. Similarly, the area of the triangle within the second strip has area 120 × (½)2 − 120 × (¼)2 = 45/2. Find the last nonzero digit of 30!. There are a number of classroom management techniques that you can use to get the most out of the lesson and the students. Lectures are kept as brief as possible and students are the drivers in the process while teachers are the facilitators of learning. Analogously, at most m marked points are single on their vertical lines. Music? Plutarch2 mentions another method of transforming triangles into squares. Thus, the sum of angles in n-sided polygon is equal to (n − 2) × 180°. Through the AwesomeMath Summer Program,1 the inspiration for this book, we've had the privilege to work with thousands of the brightest minds from around the globe for over 10 years. But that isn't the full story – it has to be an area in which you are passionate and talented – and most students won't take the time to even play video games for 10 000 hours unless they are the small handful who want to compete at a professional level. Let us denote the side length of triangle A by a. Student A said that he will participate only if his best friend B participates, too? Change the variables? FLIPPED CLASSROOM When class time is limited, teachers don't want to lose precious collaboration moments with lecturing. This is perhaps one of the biggest challenges in general as a teacher, since you cannot fake enthusiasm. Rectangles: A rectangle is a parallelogram with a right angle. So the next letter is P. The results of these projects have completely blown me away. But this is not the only reason students should pursue this level of math. Therefore, in a 12-sided figure, you have 12 vertices, and you cannot have diagonals from the two adjacent vertices, so you must subtract 3 to include your vertex and the two adjacent vertices that equals 9. What things would the parent have to ask their child to stop doing so as to move on with their day? A difficult set of pains to overcome are community and busy schedules. Some of these aspects are presented in following problems. A prime number p such that p2 divides (p − 1)! + 1. A team consists of 15 students, high school age or lower. As described in Chapter 2, math circles are math programs for middle and high school students offered on a periodic basis (sometimes weekly, bimonthly, or monthly) and appeal to students looking for mathematics enrichment and topics beyond what the schools offer. She convinced administrators to allow her to give an 80% for correctly answering 40% of the exam, and so it opened up the other 60% of the exam to ask much more challenging questions. Students who struggle more can not only be inspired, just like those who are inspired by sports athletes, but also find the confidence and support by attempting problems in a kind environment that sees the value of their contributions. The first term of the sequence is denoted by a1, the second by a2, the third by a3, and so on until the last term, which we will denote by an. In the heat of the moment, you may not write down every step to solve the problem, but you should have enough signposts to let you know where your brain is taking you. Source: Math Leads for Mathletes, Book 1. It's a function of effort, and while there are some people who seem to have a natural talent, that is only because they feel passionate about the topic and therefore put in more time. To my youngest son, Adam, for his content corrections and positive support that kept me on track and enjoying the process. Waldemar Pompe, University of Warsaw in Poland, Institute of Mathematics What Were Your Own School Experiences Like in Your Country That Contributed to Your Love of Problem Solving? Whether it was dividing property fairly, understanding the calendar, trying to bring order to the universe, or wanting to count the flowers in a garden, math has a story and history that can make learning exciting. To train educators on how to employ a problem-based curriculum in their classrooms by creating a collaborative, kind, and engaging environment where each student can be guided to be their best version. Example 8 We have the following statement: n! + 1 is a perfect square of some integer. Answer: 32 (all knights will be on white squares) Can a knight start in one of the corners and arrive at the opposite corner traveling through each square precisely once? Prove that 2T5 + P5 (where P4 is the fourth pentagonal number) is a perfect square. Powers of Two If we sum the entries in each row, we obtain powers of base 2, beginning with 20 = 1. That is in a sense more important than having the bright idea for answering the question: the right questions prompt interesting ideas! For the motivating approach, I usually start with introductory and explanatory (not necessarily trivial or easy) problems. If x is the length of the ladder and y is the distance of its bottom from the wall in the first situation, then the distance from the wall in the second situation is (y + 13.5). It's important that they are prepared ahead of time for what is being covered in the class either through watching/reading the lecture notes for the class the day before (flipped classroom) or having a study sheet provided ahead of time. This allows students to grow their critical thinking skills through exposure to practical and interesting problems. Ben Franklin LEARNING OBJECTIVES To see the beauty of magic squares, develop critical thinking skills and have fun by solving these puzzles and inspiring students to create their own magic squares. Pythagorean Theorem A square drawn on the hypotenuse of a right triangle is equal in area to the sum of the squares drawn on the other two sides of the triangle. Do there exist seven consecutive integers with sum of 130? 1, 1/2, 1/4, 1/8, 1/16 b. She participated successfully in Romanian mathematics competitions. New York: Springer-Verlag. 4.5 11. They may not have the training necessary, but they should be able to help their students know how to question well. Many problems involve consecutive even numbers or consecutive odd numbers (for example, 2, 4, 6, 8 or 13, 15, 17). And so much more! There is a world of mathematics to discover, and the playful pursuit will ignite their interest and provide them with the introspection to know their strengths and passions. We use the same idea as in problem 4: the number of zeros at the end of the factorial is the number of fives in its product. Two times the sum of a triangular number and a pentagonal number is equal to a perfect square. It can be demotivating as a teacher to constantly sell your product, i.e., a strong mathematics education. From the following picture: It is obvious that YZ2 = XY2 + XZ2 = 202 + 212 = 841 = 292, therefore YZ = 29 miles. Every type of student can play an important role in your mathematics class, and as the teacher, you want to look at every student as a collection of strengths as opposed to a collection of weaknesses that need to be fixed. (about 5–6 minutes) (about 8–9 minutes) (about 8–9 minutes) Problems 1. Up to 50% is the foundational knowledge necessary to understand the material and stave off frustration. Its initial development was motivated by problems of statistical physics. Have fun with problems: group and independent problems. Being able to see young PhD students who are passionate about their research talk about mathematics in such an engaging way connected deeply with the students. The answer is closer to being correct, but something is still wrong. I will always be grateful to my math teacher for actually convincing me that I was good at math! This process started almost unnoticeably for me and for my classmates when my teacher began telling me that some of my proofs were better than the ones she knew. Collaboration classrooms, when initiated correctly, will seek the strengths of each individual who can then bring his or her skills to the table. Example 1 Consider a polygon with vertices A1A2…An. Prove that the sum of angles in n-sided polygon is (n − 2) × 180°. How can they bring both qualities to bear so that those whom they lead will aspire to be their better selves? One can say that a magic square of size n consists of the numbers 1, 2, 3,..., n2. Please note, however, that the AMC 10/12 exams are given on the same day and have questions in common, so this student would have to take the “A” test at one level and the “B” test for the other level. Having other like-minded teachers, either in your school or district, allows for face-to-face interactions and sharing of resources. It's also important to have flexibility with the roads you choose because as you work through a school year, you may need to pivot the strategies you thought would work and adopt new ones as students' skills evolve. Math phobia. −2, −1, 0, 1, 2. Quadrants of Success As an educator, one important goal is guiding students toward success, but what does that look like? Dr. Mirroslav Yotov: When I teach problem solving, I need to have something interesting to share with the audience. Solution: 2. It provides in-depth enrichment in important areas of number theory by reorganizing and enhancing students’ problem-solving tactics and strategies. As follows, we consider representations of the unity: where the terms in the representations are not necessarily distinct. It can be tough, but an active learning environment is, well, active! That means trying to bring order to the chaos so that time isn't wasted and deeper learning can happen. Inclusion-exclusion principle for three sets. Not all schools have this luxury, and many times, truly gifted mathematicians are left to fend for themselves. For a triangle with a right angle, the square of the hypotenuse, c, is equal to the sum of the squares of the other two side: a2 + b2 = c2. Pick's theorem: Let P be given simple polygon. This can be created on the geo board in numerous ways, and when the vertices are on the grid points, it is a simple polygon. We get 12! + 1 = 479 001 601, which is divisible by 169, so 13 is a Wilson prime. According to an interview with the New York Times,3 Dr. Loh responded to a question about IMO team members from other countries training with the US team: First, bringing in the international students gives the top US students peers. Autonomy to Solve Your Problems Mastery Through Inquiry Purpose with Competitions Quadrants of Success Notes CHAPTER 5: Gains and Pains with a Problem-Based Curriculum Teachers Students Parents Notes SECTION II: Teaching Problem Solving CHAPTER 6: Five Steps to Problem-Based Learning Start with Meaningful Problems Utilize Teacher Resources Provide an Active Learning Environment Understand the Value of Mistakes Recognize That Everyone Is Good at Math Notes CHAPTER 7: The Three Cs: Competitions, Collaboration, Community Competitions Collaboration Community Aspire to Inspire: Stories from Awesome Educators Further Reading Notes CHAPTER 8: Mini-Units Relate/Reflect/Revise Questions Roman Numeral Problems Cryptarithmetic Squaring Numbers: Mental Mathematics The Number of Elements of a Finite Set Magic Squares Toothpicks Math Pick's Theorem Equilateral versus Equiangular Math and Chess Area and Volume of a Sphere Notes SECTION III: Full Units CHAPTER 9: Angles and Triangles Learning Objectives Definitions Angles and Parallel Lines Summary Note CHAPTER 10: Consecutive Numbers Learning Objectives Definitions CHAPTER 11: Factorials! Learning Objectives Definitions CHAPTER 12: Triangular Numbers Learning Objectives Definitions Notes CHAPTER 13: Polygonal Numbers Learning Objectives Definitions CHAPTER 14: Pythagorean Theorem Revisited Learning Objectives Definitions Pythagorean Theorem Rectangular Boxes Euler Bricks Assessment Problems CHAPTER 15: Sequences Learning Objectives Definitions Introduce a Geometric Progression CHAPTER 16: Pigeonhole Principle Learning Objectives Definitions Note CHAPTER 17: Viviani's Theorems Learning Objectives Definition Note CHAPTER 18: Dissection Time Learning Objectives Definitions CHAPTER 19: Pascal's Triangle Learning Objective Summary Notes CHAPTER 20: Nice Numbers Learning Objectives Definitions Index End User License Agreement List of Illustrations Chapter 7 Figure 7.1 The shortest path between two points on the plane is the segment co... Now what? It also lends itself well to being chunked; namely, you can reason out the individual steps, or chunks, that lead you to the solution. Math education must be more than a series of easily obtained answers (exercises) (e.g., find the perimeter of a square, training students to do what a computer can do better). To my husband, David, whose ideas and insights have added value not only to this book, but to our family for over 25 years. Take the time to notice the good things whenever they happen. Students are a microcosm of what took ancient civilizations thousands of years to understand. Recognize that everyone is good at math! Start with Meaningful Problems In this section, we will cover the following: What are good problems? At first, since all the angles of ABCDE are equal, all have the measure of 108°. Count multiple events. One of the problems was to find the mistake in a proposed “solution” to a given problem dealing with abstract algebra. Having bounds or limits/measurable. Yes, they are equal, and the sum is 72. They have the possibility to move to the right and up. Today, schools and teachers tend to fall into one of two camps as either fans of the clumping method (having those with higher talent grouped together) or the scatter method (having students of varied skill levels mixed together). We have |A| = 118 − 9 = 109. Dissect a 4 × 9 rectangle into two pieces, which could be assembled to form a square. It helped that I had a precalculus teacher, who was also our competitive mathematics coach, who was a visionary. A polygon is equiangular if all of its angles are congruent. 2. Here are a few examples: Growth mindset. Which number is greater, 7! or (1 + 2 + 3 + ... + 100)? Now, we have 5 pigeons (the numbers) and 4 holes (the intervals). When students struggle, that is when they grow and learn, and when you struggle in a supportive environment, you are more willing to be vulnerable and have false starts on your quest for the solution. Fractions, decimals, and percentages. Active Problem-based learning, because collaboration is such learning a large part, creates an active learning environment where students ask questions and work willingly on the problem sets with peers and their learning facilitator (the teacher). Writing problems that others would enjoy solving brought joy as well, and this transferred to music, where he would compose songs and loved writing lyrics. If he read a book or story, he would incorporate the aspects he liked the most into whatever story or game he was building or even go so far as to write fan fiction and continue the story even after the original ended. In the figure below, lines L1, L2 are parallel. Definition of congruent squares Suggestion. One path leads to certain death and the other to freedom. Treating encounters with the mindset that the person they are engaging with can be a future colleague and/or potential letter of recommendation is a mindset that should be adopted. What is the relationship between the line lengths of the similar triangles? When things are going well, we sometimes forget to stop and be grateful, so make it part of the routine to look for the positive and take note of it. The problem I have in mind is one I tried to solve some 30 years ago and my first attempt to do that turned out to be wrong. So, collaborative work is important, but it needs to either be supervised or in reasonably sized groups. Only 1,600 pounds for such an enormous gorilla? Focus So many students today are diagnosed with some sort of attention deficit issue or may not be getting the time during the school day to burn excess energy, which causes problems with focus in large classes with lots of distractions. You can talk with the teacher or peers if you need more guidance. I fell in love with that way of thinking: how can this be done faster, easier, better? Because 10 divides 5! + 6! + • • • + 100!, it follows that they have 0 as the last digit. As we noted in Section 1, it's all about the journey with your student by being their guide and mentor so that after crossing the threshold to learning, they are changed deeply by the experience. He published over 700 papers and 50 books. thinking. Continuous A problem-based curriculum provides challenge levels challenge for all the students in the class (scalable) and since the focus is on process and not outcome, problems can be harder. This is an opportunity to make connections, explore, and find the story of mathematics. Example using the chart below: N Tn Sn Pn 1 2 3 4 5 6 1 3 6 10 15 21 1 4 9 16 25 36 1 5 12 22 35 51 So if we pick the fifth number, you would solve it the following way: Algebraic proof: 11. Triangulation. Since 2a + k > k + 1 and 2a + k − (k + 1) = 2a − 1, we find that k + 1, 2a + k are factors of 200 with different parities. These kids are immersive learners and want to collect mathematical techniques and tricks. Now consider a line AB that passes through P and P is in between A and B. The math competition community is a kind and supportive group where students and teachers alike enjoy discussing problems. One of them is the following: 4. It's not about being excellent at specific concepts, but instead, showing the students to what they are connected and how they will evolve. When you read the solutions, try to reconstruct the thinking that went into them. Then x + (x + 1) + (x + 2) + (x + 3) + (x + 4) = 90, which implies that x = 16. Metroplexmathcircle.org. Angles and Parallel Lines Assume you have a point P in the plane. This implies that n = 6. They figured out the cost, did research on what other universities offer and how that has affected graduation time of their students, used (basic) mathematics to analyze cost and benefits, and put up a fantastic presentation of why this needs to happen. Assign different group leaders for a collaboration team so students learn to listen to each other and experience different types of group manager approaches. Write the following Roman numeral numbers in Arabic numbers: LXII, XV, LXXXIX, CCLVII, DVIII, DLXXXVI, MDCXXXIII, MMVII, MCMXCVIII Answers: 62, 15, 89, 257, 508, 586, 1633, 2007, 1998 Challenging Let N be the greatest positive integer that can be expressed using all seven Roman numerals I, V, X, L, C, D, and M, exactly once and let n, be the least positive integer that can be expressed using these numerals exactly once each. The autonomy to take the reins and guide their learning inspires them to excel. The above magic square is known as the Lo Shu square. CHAPTER 7 The Three Cs: Competitions, Collaboration, Community To have an effective and efficient problem-based learning (PBL) classroom means engaging in mathematics competitions in a collaborative environment and being a part of a larger community for support, both for the teacher and the student. It was more work, but the class saw greater improvement over a shorter period of time. As an associate professor in Mathematics at Youngstown State University, I have a lot of very different opportunities to work with students in problem solving. If we try H = 3, then D = 6, and the only possible value for C is C = 8. Solution: 1. Ask leading questions to a problem so that the process slowly unfolds in front of the students and they see the joy of figuring things out. It is student-centric as opposed to teacher-centric. Example 4 Dissect a regular octagon in pieces and assemble them to form a square. Is it the wording, critical thinking, foundational knowledge? When seeking other teachers who value PBL, you have a number of choices: Local peers. When exploring a range of topics, this is not something to be done alone. Titu Andreescu and Branislav Kisacanin, Math Leads for Mathletes, Book 1, page 44, part of “Fun Sequences” (Providence, RI: American Mathematical Society, 2014). This number is denoted by |A|. (Purple Comet Math Meet Contest 2019, created by Titu Andreescu) Solution: 222 The number N is expressed by listing the numerals in decreasing order of value: MDCLXVI, which represents 1666. Students will see that what they are learning carries them past their time in school and will benefit them throughout life. Show them multiple approaches to solving a problem or proving a theorem. Apply computational analysis to language and speech for linguistic phenomena. When we allow students to work toward mastery instead of grades, then the journey becomes about the process and not the outcome. Polygonal numbers are the number of vertices in a figure formed by a certain polygon. Charles Mingus Problem solving requires creativity to reach a solution, as there isn't always just one clear-cut approach. Rather, the book gradually builds students’ number-theoretic skills and techniques. When you immerse yourself in the creative process of either rewriting problems to make them more meaningful or creating problems of your own, it opens another door for how to think about mathematics. Here are sample mission statements that could be created by your students depending on their maturity levels; the first is more complex and the second more simple: This mathematics class rewards the process over the outcomes so that each student can grow mathematically, be innovative and curious, and collaborate together to solve meaningful problems that will help them succeed in school and prepare them for the world ahead. Support If parents don't understand the importance or necessity of a problem-based curriculum, they may feel the effort is a waste of their student's time. Example using the chart below: N Tn Sn 1 1 2 3 3 6 1 4 9 4 10 16 5 15 25 So if we pick the third number, it would be as follows: 2T3 + S3 = 2(6) − 9 = 12 − 9 = 3. Busy Problem-based learning requires slow and deep schedules thinking for which today's overscheduled student doesn't have the time nor do many families. CHAPTER 11 Factorials! Learning Objectives Students will solve problems involving factorials. CHAPTER 17 Viviani's Theorems Learning Objectives In this section, we will study different versions of Viviani's theorem.1 Viviani's theorem is a geometric fact that's challenging to prove but easy to visualize. The grand breakthrough came after a problem was solved only by me, which caused my teacher to pull me out of my literature class and asked me to explain my solution to the students in another math class! This was a public recognition that elevated my confidence and made me respected by all my classmates. Explain the rules. Time Some students require more time to process and absorb information, so the time-intensive nature of solving difficult problems can be an obstacle for them. 12. Below are some example problems from the book Math Leads for Mathletes, Book 2, centered on least common multiple (lcm). Sequence of numbers that are of the form 3k + 1. 3, 1, 2, 8, 3, 1, 3, ... 12. This is an incredibly difficult exercise, so students might need to consult with their parents to help determine what drove them when they were younger, but it's worth the time so that your class can be designed with the students who you are teaching in mind. Teaching problem solving is, to me, like teaching someone to throw a spiral with a football: something students have to do by trying, not by watching. This implies that n ≥ 1011. In the quadrilateral ABCD, E is the midpoint of AB, F is the midpoint of BC, and G is the midpoint of AD. Hence, θ = 18°. Further, having the accountability to meet with peers and discuss interesting problems, either ones you've discovered or ones you want to create, ensures that the process will be a priority in your life and make you more open to new ideas as they come along. Another part of the process is that sometimes the problems you write are just unsolvable, or you don't yet possess the mathematics background required to solve what you create, and that's okay! Give yourself the grace to understand you won't know how to do everything and make sure to enjoy the creative exercise. Following are some things to keep in mind when devising your roadmap. In the former case, a = 9, k = 7, and in the latter case a = 18, k = 4. There was (and continues to be) a lot of trial-and-error in finding active learning methods that worked for me and my classes, and there are a multitude of active learning techniques that could work for others. Math competitions probably were the first thing in my schooling career that brought intrigue to problem solving. This sometimes helps them realize that in order to solve the initial problem, they must exploit somehow the part of the hypothesis I dismissed at the previous step. For an equiangular n-gon, each angle is 180° − (360°/n); this is the equiangular polygon theorem. How many sides does the polygon have? Thus, the perimeter is 11 × 3 + 3 = 36.2 Have your students close their eyes and start doubling numbers in their head slowly as a way to create rhythmic breathing (each breath should take five to six seconds). In other words, the process once again is more important than the outcome. Solution We see that for n = 1,2,3 the statement is not true. Students can create other n-gons and measure their angles to prove the equiangular n-gon theorem. Even the terms used for learning piano and learning mathematics are different: Students play piano and work on math problems. Collaboration. Kluwer Academic Publishers But wait, if you just add the pets, you double count the students who own two or more, so they need to be subtracted out: 42 − 2 − 4 − 6 = 30. There is a large difference between knowing and understanding. Let the diameter of each chocolate in the smaller box be d. As a student leader, they need to have the confidence to take on the task, so how do you get them over fear of leading? We need students who can do more than solve mere exercises for a check mark; they need to be able to tackle difficult problems and also be able to notice problems worthy of solving by seeking patterns, reframing information, and asking the right questions. JOURNEY OF DISCOVERY AND THE IMPORTANCE OF RISK With a human-centric, individualized approach to education, it's important to ensure that the student is the hero of their own story. The square of the main diagonal of a rectangular box is D2 = a2 + d2, where d is the diagonal on the face of the box with sides b and c. Dudeney and first published in the July 1924 issue of Strand magazine. It should be clear, concise, and connect with all stakeholders. To contact JosseyBass directly call our Customer Care Department within the United States at 800-956-7739, outside the United States at 317-572-3986, or fax 317-572-4002. There was not too much thinking going on. How many handshakes occur? Notice that, according to this definition, any parallelogram is a trapezoid as well. Because 360 = 23 × 32 × 5, it has (3 + 1)(2 + 1)(1 + 1) = 24 divisors, including 1 and 2, the answer is 22. For example, you can feel better about a team's progress when you have the perspective to see how they are improving over the course of the class (e.g., compared to how they were last week). It's this collaboration and discussion where growth happens and the students are given agency (ownership) over their learning process. The manipulation of large numbers using their prime factors is a fundamental exercise in number theory and reveals new ways to think about mathematical operations. In other words, there isn't one absolute formula for success, but rather, there are various qualities that make a leader worth following. So which numbers are divisible by 5 in 25!? Figure 7.4 According to Bolzano's theorem, there exists a point M in which two paths meet. The best way to assess best practices is through feedback loops. Be exposed to problems in the area of discrete math, such as number theory or combinatorics, which have many applications in today's world. When did the student seem the most intense? Students are given the time and tools they need to solve the problem in their own way, they are given the support to follow through on their ideas, and they can be given problems outside their grade level to see the wonders that lie ahead. Summing the rows reveals powers of 2. What feedback did you get from the students? For example, machines were historically designed around engineering principles (e.g., faster cars or multipurpose machines). Then there must be the ability to choose a path and have the talent and skills to make a difference. She would connect with people over ideas (as opposed to emotions) and felt that those relationships were the most worthwhile. The randomization can be as simple as “talk to the student to your right” or “all students born in May, work together,” etc. How do you think that as a teacher, I would choose to test students' knowledge? The definition of an Euler brick in geometric terms is equivalent to a solution in integers to the following system of equations: where a,b,c are the edges and d,e,f are the diagonals. I got my education there, too. One could argue that chess is a nonzero sum game even though a “versus” type by its very nature is zero sum. Same strategy as in the previous problem. Provide an exercise of visualizing a sectional view of a sphere. Strengthen their relationship with the teacher and add value to the class. There are so many wonderful places an enriching math education can take you. The lcm (6, 8, 24, 30) = 2(3 × 3 × 5) = 120. Today's positive shifts in education with collaborative problem solving, growth mindsets, and design thinking, as mentioned before, are student-centric for the purpose of inspiring students and defocusing outcomes. That is why this is not just a declaration of perspective each teacher will have, but instead, a starting point to get the educator to think about the value a problem-based curriculum can provide for their student. While it will be a challenge to keep up, your math education is stronger, and you can learn new things in the process. Hence there are three solutions to the problem. The class is a team, and working through errors together offers a great teaching moment where everyone will benefit. A property of angle bisectors of a triangle: Every angle bisector divides the side it intersects in two segments proportional to the other two sides of the triangle. A sequence of terms in which the ratio between any two successive terms is the same. He tells you that 20 students have a cat, 15 students have a dog, and 7 students have a bird. This may be an interesting and unexpected idea, or a problem with interesting formulation, or a series of problems that reveal an interesting aspect of math objects. This is contingent on having a classroom where trade is open and welcome. Yes, my teaching methods have evolved over the years. the correct solution. Thus, AG = GB. It follows that the sum of the shaded areas within the four strips is (60 − 15/2) + 45/2 + (60 − 75/2) + 105/2 = 150.2 Because of these constraints, people will reach the conclusion that mathematics is dry and formulaic, but anyone that has ever delved deeper beyond the typical classroom approach to math understands how these constraints free the mind and lead to amazing discoveries. Calculate 11S24 + 2S5. Praise your child for asking any question, even if it is a simple one, since this is the path to discovery. Figure 7.5 A shorter system of paths consists of the segments AM, BM, CM, and DM. The number of dots in a triangular array. Unfortunately, in today's education system, students are expected to check the boxes of a rubric or standardized test rather than question and think outside the box. The students will learn to represent these numbers algebraically and find patterns in consecutive numbers. There are no negative consequences if you get the problem wrong. Therefore, the last two digits of 72020 are 01. Replace each number by its Roman numeral equivalent and the sequence reads FIVE SIX SEVE, so the next letter is N. Supplying teachers with a treasure trove of problems for students of all levels, which aids lesson planning and saves time. Tell me and I forget. At the AwesomeMath Summer Program, we encounter these exceedingly bright kids all the time and have the faculty necessary to either increase their class level or pivot the class material quickly to reach the needs of even the most gifted problem solver. Viviani's theorem. Find the number of diagonals in a dodecagon (12 sides). Note that 5! is divisible by 10; hence, the digit is 0. The answer is 150. It is clear that in the nodes A1, A2, A3 and B1, B2, B3, there is only one possible path to choose. Each circle has area 62π = 36π. Full of the philosophical groundwork, expert insights, and plenty of practice problems, Awesome Math: Teaching Mathematics with Problem-Based Learning is a must-read for any Math or STEM educator concerned with the relevance and joy of a beautiful and expansive discipline.” Ben Koch, Co-founder and CEO, Numinds Enrichment “Awesome Math makes a strong case for ditching rote memorization and turning to collaborative problem-solving and mastery-based learning instead. Each person shakes one another's hands, but it counts double because it occurs separately from each person's point of view. This autonomy gives them the choice to be their best version and develop understanding for where their particular set of skills adds the most value to various problems and various groups. You'll be combating a culture where it is okay to give up on math; therefore, the connection to real-life problems is key. Prove Viviani's theorem for equiangular polygons. Dr. Branislav Kisacanin, Computer Scientist at Nvidia Corporation What Were Your Own School Experiences Like in Your Country That Contributed to Your Love of Problem Solving? Students can also write a math sequence of their own, including the formula to determine the next number. If we consider Pascal's triangle modulo 2, we find the following figure that is similar to Sierpinski1 triangle. The numbers are the number of letters in the names of the numbers (one, two, three, and so on). Figure 7.5 A shorter system of paths consists of the segments AM, BM, CM, an... Of course that increases the level. Example 3 In the following figure, ∠ACB = 40°. We learned because we took things apart, got our hands dirty, and tried different approaches. In Section II, Teaching Problem Solving, you will have access to curriculum for a full class period, 45– 55 minutes, as well as mini-units (Chapter 8), 10–15 minute exercises comprised of hook problems, which are intended to get the kids thinking deeply while being scalable to various skill levels. It is possible to go backward from a square to a polygon, and therefore transform any polygon into any other polygon of the same area, with a square as an intermediate step. The entries of the fourth column are tetrahedral numbers, or triangular pyramidal numbers. 31 = 2 × 11 + 9, we showed before that 11 is nice; thus, 31 is also nice. Example 4 How about this magic square? A large factor in the success of individuals is the people with whom they surround themselves. Advanced mathematics programs teach concepts not offered in today's schools. The next number not to contain the letter E is 2000. Our inner cores stay remarkably the same. Of course not. That is why taking intellectual risks in a human-centric classroom is so important. Each angle of a polygon is equal to k, where k is a whole number. Solution Since triangles ACD and CBA are similar, we find that ∠ACD = ∠CBA, ∠CAD = ∠BCA, and ∠ADC = ∠CAB. See Grades Grace: in learning environment, 27, 29–30; under pressure, 98; skills from, 78–79 Grades, 69–71 Grid, 165 Groups: activities for, 150; clarity for, 51; for collaboration, 10, 50; competition for, 15; math circles for, 16–18; in PBL, 126; play for, 7 Growth mindset, 35 Gudder, S., 14 Guessing, 223 Guidance, by teachers, 56 H Hardy, G. The next state would be Kansas. You want to constantly be in a “what if” mindset, and one way to document your journey of questioning is to keep a notebook handy where you can record your questions and thoughts. So, we must calculate the following sum: Note that (n(n + 1))/2 = (n2 + n)/2. Show that 10, 20, 22, 24, and 34 are nice numbers. The diagonals of a parallelogram intersect at their respective midpoints. Students are remarkably good at knowing their own strengths, and it was never an issue for them to select a difficulty level, especially when given a handout of the problems at the beginning of class. You're Reading a Free Preview Pages 180 to 183 are not shown in this preview. know? My teachers were not only teaching us their subject, they were also trying to detect our strong abilities and to encourage their further development. What are four consecutive even numbers that have a sum of 92? Each following terms is 3 times the previous minus 1. Malcom Gladwell To attain elite status and truly be good at something, Gladwell contends that you must practice early and often so that you can hit the magic “10 000 hours” number. The learning environment for the game is critical to bringing out the best in the players and the rewards are range, rigor, and resilience. And while the cargo cult might seem to have been doing everything needed to bring bounty from the sky, they weren't truly understanding what was actually happening. If T is a triangular number, then 9T + 1 is also a triangular number: a. Find the length of BE. Many thanks also to Pete Gaughan, the content enablement manager for this project, who helped us every step of the way to create a quality publication. Note that congruent triangles are similar triangles, but the converse is not true.1 Also, because we call these triangles similar, they look the same and one triangle is a rescaling of the other triangles by some constant k. Having students develop a mission statement and core values list will help set the proper mindset right from the start that kind collaboration is the goal for every class. Since 2009 = 72 × 41, all divisors of 200910 would be of the form 7a × 41b where 0 ≤ a ≤ 20 and 0 ≤ b ≤ 10. So

15, and the area is equal to K = 15 + (15/2) − 1 = 22.5. ADDITIONAL PRACTICE Using a geo board, students create different polygons and compare how the interior points affect the overall area. If P6 = 51, T6 = 21, and S6 = 36, then P6 + T6 − 2S6 = 51 + 21 − 2(36) = 51 + 21 − 72 = 0. As it happens, the only magic square that we can make using those numbers is the one we talked about in the first figure. Can you formulate a more general question? But luckily, I was by then aware that there is a whole mathematical universe out there full of mysteries that can be discovered using rigorous thinking. PBL hinges on truly understanding the underlying complexities and beauty of a problem over quickly reaching a solution. Then, could you reword or rework the problem so it captures more of the list? Prove that there is at least one number that appears at least 17 times. Competition math training challenges students and prepares them for discrete mathematics, college admissions, and critical thinking. Basic Algebraic Inequalities 1. Triangles FDC and AEC are similar. Factorials represent the number of ways a set of objects can be arranged, so they appear frequently in combinatorics. Are they equal? Having accessible manipulatives, hands-on projects, and real-world circumstances as a part of the problem-solving process will make the material relevant, authentic, and useful – this is more than can be said for video games! STUDENT GAINS Critical thinking Multi-step problems that can be solved in more than one way train students to think critically in order to work them out. The following questions will help you and your students process what has been covered and get the most out of the units. The midline theorem for trapezoids: The midline of a trapezoid is parallel to the bases of the trapezoid and has length the arithmetic mean of the lengths of the bases. Yes, but no!!! (as famous Bulgarian journalist Petko Bocharov used to say). Find the largest value of a + b + c + d. As they learn more, they will be able to find more connections and patterns that they will see are important to the process as a whole. Kathy began her own enrichment school, Eudaimonia Academy (2006–2012), where she coached math teams, taught a philosophy/creative writing course, and co-led speech and debate teams. Name the fear. Build the shortest road network (system of paths) connecting the cities. Do they think about similar problems? Problem-based learning approaches education with a deep respect for the value, abilities, and strengths of each student by raising expectations beyond the standard and providing guidance in a supportive environment. Not only in the math club, which prepared students for the Olympiads, but in my math classes in general, there were always opportunities to think in different ways and to solve hard problems with few time constraints. In a rectangle, diagonals are congruent; for example, AC ≡ BD ≡ A′C′ ≡ B′D′ = f. Encourage the leader to ask students to come up with a plan ahead of time as a group for tackling this unit. Then, there is at least one triangle to pass through two of them. With a problem-based curriculum, there is no ceiling on learning and there is ample depth and breadth of subjects to keep students challenged throughout their lifetime. When students drive their own learning by working to understand the lecture, retention of material and confidence can be increased. Hence, (n + 1)! − n! = 6n. For me, that community is the MAA Project NExT, which is a fellowship program in the United States for new faculty in higher education, though similar programs and Math Teacher Circles can be found for other regions and school levels. This semester, a group worked on getting a commuter center at our university. 5, 7. Further, when the focus is on each child becoming their best version, they are free to learn in the way that is best for them. We can place the bishops in the following manner: A1, A2, A3, A4, A5, A6, A7, A8, and H2, H3, H4, H5, H6, H7. Calculate 12T23 − 7S9. Furthermore, a collaborative and problem-based approach gives younger students a more accurate impression of what higher-level math entails. Before life became about homework, video games, social media, and/or meeting expectations, how would students spend their free time when they were around seven or eight years old? Prove that on some pair of consecutive days, the computer was used at least 15 hours. Suppose that the vertices of a polygon all lie on a rectangular lattice of points where adjacent points on the lattice are a distance 1 apart. As the hero of their problem-solving journeys, students will go through these transformations multiple times, and their mathematical knowledge will deepen and transform. The guard that always lies would, well, lie and not tell the truth. If two triangles have two corresponding pairs of angles with the same measure, then they are similar. Kathy is the marketing and communications director for the AwesomeMath organization. Then, there are lessdefined roles that come about more organically. Be careful: 2x + 1 is not even. 3. Further, 74 = 2401 and 78 is 2401 × 2401, which ends in 01 (5 764 801). The pigeonhole principle, or Dirichlet's box principle, usually appears in solving problems in algebra, combinatorial set theory, combinatorial geometry and in number theory. Solution Example 2 Is 2 a nice number? Conversations can take several forms – from assigning scaffolded problem sets to be solved in groups, to giving students 30 seconds to briefly explain an idea or definition to their nearest neighbor – and are easily adjusted to fit different classes and classroom environments. I suspect nothing has changed for the better since that time. Other arithmetic sequences, such as consecutive even or odd integers, have their own emergent qualities. If the lesson for the day was on concepts in algebra, students who have that strength would sit at the Bach table and those who may need more guidance would sit at the Beethoven table (they would self-select). As an adult, she enjoys research, education, and cross-discipline pursuits. Solution Since 7! = 5040, T7 = 28, 3! = 6, and S5 = 25. Since in any equilateral triangle, the largest distance between two points is equal to the side length, we find that the side length of each equilateral triangle is greater than or equal to a/2. Below is an algebra problem that students can also use logic to figure out. It may mean research on your part to ensure you believe in what you are selling. A reflective thinker may need to see problems a day ahead so they can have the necessary time to process what they are reading and employ their powers of thinking deeply and slowly. The thing you don't know and can't control is the student catastrophizing about possible outcomes that aren't rational. A 30- or 45-minute class period can create a constraint to problem-based learning, especially if it is a large class. Breathe in 1, breathe out 2, breathe in 4, breathe out 8, breathe in 16, breathe out 32, breathe in 64, etc. What exactly is problem solving? Of course, if I had tackled the problem with first inquiring what was known about it, I might have never gone along this road. What Were Your School Experiences Like in Your Country That Contributed to Your Love of Problem Solving? The area of the triangle within the third strip has area 120 × (¾)2 − 120 × (½)2 = 75/2, and the area of the triangle within the fourth strip has area 120 − 120 × (¾)2 = 105/2. Problem solving is the strategy, and math competitions are the vehicle to train your math class to be stellar thinkers. I fell in love with geometry, which was more challenging than algebra, which I was doing easily without too much thinking. To obtain the third polygonal number, extend by two the sides of the polygon from the second polygonal number and complete the larger polygon by placing vertices and other points where necessary. Moreover, since G is the midpoint of AD, we find that DG = AG = BG = CG. The series of numbers 20 through 28 with their digits grouped in threes. These are the initial letters of the US states, listed alphabetically (Alabama, Alaska, Arizona, Arkansas, and so on). In order to find |A∪B∪C|, observe that 5 × 7 × 9k ≤ 2009 is equivalent to k ≤ 6; hence, |A∪B∪C| = 6. 10.5 4. It is so hard to find this type of problem, particularly ones that are a good fit for the high school mathematics classroom. The “Polygonal Numbers” unit goes by more quickly if students work together, but it can be difficult to divide the work appropriately. Malcolm Gladwell became famous (or infamous in some circles) with his 10 000-hour rule as stated in his book Outliers.2 Ten thousand hours is the magic number of greatness. five consecutive numbers 3. Example 5 Simplify the following expressions: 1. Also, we will let the face diagonals be AB′ = d, A'D′ = e, B'D′ = f. These connections, explorations, and stories bring depth and wonder to the study of mathematics and help engage all students in the class by capturing their curiosity, as opposed to setting up situations where they, instead, feel ranked, such as by timed tests, worksheets, and rubrics. What value does the class add to each student? You're Reading a Free Preview Pages 8 to 13 are not shown in this preview. Assume without loss of generality a < b < c < d. Can they think of a simpler example? Their sum is equal to mn + ((n − 1)n)/2 = p, where p is a prime. Triangular numbers form an interesting sequence. This topic was loved by all the participants, for the following reasons: Relevance – he connected the topic with the proof that won the Millennium Prize. Now, in virtue of the triangle inequality, we can reduce our system of paths to the diagonals of the square (Figure 7.6), which implies that this is the shortest system of paths, right? Beyond the leadership role, there are other roles students can take ownership of to ensure the group is effective. Then its area is equal to K = I + (b/2) − 1, where I equals the number of inside grid points, and b is the number of grid points on the sides of the polygon, and K is the area of polygon P. Various education backgrounds and educational expectations Various multigenerational families living together And so many more combinations All these types of families mean that the pains and gains from a parent perspective with a problem-based curriculum can be just as complex as the lives of the parents. Studying the biographies of mathematicians can make math relatable as well. The team leader has to make sure the students' argument is sound, which means listening carefully to everyone's ideas. Students do not listen to each other, insist on their ideas being listened to, and even some of them become lazy and do not work at all, waiting for somebody else to do the job. Then the volume of the chocolate in the first box is V1 = 8(4/3)π(d/2)3 = (4/3)πd3, while the volume of the chocolate in the second box is V2 = (4/3)πd3. A typical black and white cow weighs that much! A hippopotamus can weigh up to 4,000 pounds. What Is Your Personal Approach to Problem Solving? Set of whole numbers and their opposites (negatives). If there are no ideas, I suggest something to help the students move off an idle point. 1, 3, 6, 10, 15, 21, ... 8. Utilize Teacher Resources Having the right tools for the job can make work more efficient as well as pleasant. Solution We have (7 − 1)! + 1 = 6! + 1 = 720 + 1 = 721. You don't want to list specific items, e.g., “This algebra class seeks to provide understanding in the areas of patterns, linear equations and inequalities, statistics, exponential equations, and quadratic functions,” but instead describe what is the common thread that unites all these areas, e.g., critical thinking, logic, creativity, etc. Visualize the face and main diagonals of a rectangular box. The annual ARML competition takes place during the Friday and Saturday following Memorial Day. They may contain some ideas that did not occur in your solutions, and they may discuss strategic and tactical approaches that can be used elsewhere. The solutions are also models of elegant presentation that you should emulate, but they often obscure the tortuous process of investigation, false starts, inspiration, and attention to detail that led to them. You're Reading a Free Preview Page 83 is not shown in this preview. Of course, there are online forums for mathematics competitions where students can tap into a global community with shared interests. The problems in this unit can all be rephrased in this way, by demonstrating that the claim in the problem is akin to sorting pigeons into holes. They need to think in terms of their younger selves (seven or eight years old). Solution: The total area is 45, so the area of each square is 9, and therefore the side length of a square is 3. Thus, we must have N = 0 and E = 5. If 4 ≤ n ≤ 20, determine the value of n. It is also valuable to think about generalizations of the problems we managed to solve together. The first thing to be understood is that you are not a judge of work that is either correct or incorrect, but instead, you are a facilitator on the path of discovery and truth. Hopefully, someone on the collaboration team will make the connection that there are 366 days in a leap year, and once that connection is made, then you will think in terms of how many months in a year have 29, 30, and 31 days, respectively. Because all angles of an equiangular polygon are equal, each angle is equal to (n × 180° − 360°)/n = 180° − (360°/n). CHAPTER 8 Mini-Units These mini-units are printables that you can use for a 15–20-minute problem-based learning (PBL) lesson. Again, these are two qualities that seem similar, and yet one can derail the task at hand if not managed properly and the other has the ability to bring positive change to a group as long as the scope isn't too narrow. And when he invited me to do this project, I again wanted to learn everything about math! So, as I said, I was lucky with my math teachers. Concise. Example 2 The numbers 16, 17, and 18 are consecutive and add up to 51. Hence, by the pigeonhole principle, at least one horizontal line contains more than one marked point. Because stakes have been raised as far as going to college, college admissions, and false metrics (e.g., grades, GPA, standardized test scores), students don't want to take intellectual risks. The next letter is E. H can only be 3, 6, or 8, because otherwise either H or D will take a value which has already appeared before. Steiner Trees in Industry, Handbook of Combinatorial Optimization, 5e (eds. Then, it is easy to deduce that PD + PF + PE = h. So how do you find those core traits for your students? See Play G Gains: from learning, 57; for parents, 68; for students, 62–63; for teachers, 59 Galois theory, 145 Game theory, 7, 11, 18 Games, 111, 114–116 Gaps, in skills, 60, 63, 66, 68, 70 Gardner, Howard, 30 Geometry, 245; angle bisectors in, 91; case studies on, 213; geometric progressions, 221–226; isosceles trapezoids, 90; knowledge in, 131; parallelograms, 89; quadrilaterals, 90; rectangles, 89; rhombi, 89; right triangles, 90; similar triangles, 90–91; squares, 90; Theorem of Thales in, 90; trapezoids, 90–91; triangles midlines, 90. If you were at the Beethoven table and had a question on how to solve a problem, you would ask someone at your table and/or someone at the Mozart table. Solution: Pick's Theorem OVERVIEW This theorem was proved by Georg Pick in 1899. Children are not outcomes and need to be guided by a great educator to think critically and creatively. Parents who haven't been exposed to either discrete mathematics topics or a student-centric approach to problem solving may feel illequipped to provide the support necessary for their student. Thus, the required area is 3 × 36π − ½ × 36π = 90π. Hence, T = 8 and X = 4. Figure 7.12 A second distinct solution with symmetry. When Randy Pausch gave his Last Lecture at Carnegie Mellon University,2 he explained that the moment someone lowers their expectations for what you can accomplish, they've stopped caring about you. Solution Example 4 Let a = 2 be the initial term and r = 5 be the ratio of a geometric sequence. Find a1,a2,a3,a4,a5. How long do you need to be around peers to count as socialization? See American Invitational Mathematics Exam Aldrin, Buzz, 55 Algebraic concepts, 199, 245 Algebraic identities, 87–88 Algebraic inequalities, 89 Algebraic representations, 187 AMC. One is as follows: Label the 2 × 2 unit squares A, the 3 × 3 unit squares B, and the 6 × 6 unit squares C. So, their corresponding sides are parallel, as shown below. Hence, by comparing the above relations, we find that d1 + ... + dn = n × D. It helps students feel more comfortable making mistakes, and by doing so, learning by experimentation. This may mean providing outside resources such as online classes, videos, books, and/or tutoring. Make the time for this exercise in every class. When interviewing teaching assistants for the AwesomeMath Summer Program, one of the applicants was a twotime gold medalist for the US Team that competed at the International Mathematics Olympiad. As operations director of the AwesomeMath programs for the past 12 years, Alina has been integral in every facet of creating the opportunities/resources that fulfill the mission of providing enriching experiences in mathematics for intellectually curious learners. A simple estimation shows that 125! has 25 + 5 + 1 zeros at the end because there are 25 numbers divisible by 52, and one number is divisible by 53. Clearly, n = 1 and n = 2 satisfy our condition (1 + 2 = 3). It follows that O = 9 and W = 8. After finding a creative solution to that problem I was hooked. Let b1 = 2a1, b2 = 2a2, ..., bk = 2ak, bk+1 = 2. Source: Purple Comet Math Meet! Contest 2019. Solution This example requires a subtle selection of holes. Finally, it follows Problems 1. The sum of all angles is n × 135°. We all have different learning styles. In this template, the hero has a call to adventure where she must decide whether or not to cross the threshold to the unknown. Figure 7.8 Now consider how to minimize the sum AM + DM + MN + BN + CN, where M and N are points inside the square ABCD. The shaded square whose sides are diagonals drawn to these division points has area 13. That is, GCD is isosceles, too. 2. MathWorks Math Modeling (M3) Challenge is a contest for high school students in 11th and 12th grades. Then the area of polygon is equal to ½D × a + ... + ½D × a = (n/2)D × a. There will still be the need for proper planning and classroom management skills; however, the mini-units are a great way to get started with a problem-based approach as well as gain the experience necessary to bring this effective method to life in the classroom. Selection to the USAMO is based on the USAMO index (AMC 12 Score + 10 × AIME Score). When you're in a mentorship role, it's never one-dimensional and it's never just one topic. The threshold to adventure should be an interesting problem or challenging concept that is worth pursuing. Some may want to compete in mathematical Olympiads, but others may have talents better suited for mathematical research. Rectangles, including the squares, are the only equiangular four-sided figures. Once step 2 is complete, it's time to devise a strategy. Our diversity is what makes us all better and broadens our worlds. We are ready to study the more general cases. Speaking of my high school experiences contributing to my love of math and math problems, I have to confess that my first year there was also very, very important. The Pascal's triangle has axial symmetry, that is, 3. A map of engagement that will help a student truly excel in their education is one where parents guide their students on how to ask the right questions about a problem(s). This has been an invaluable tool at the AwesomeMath Summer Program where the core values are: Kindness Community Independence Fun Whenever a student strays from the core values of the class, they can be asked, for example, “Are you demonstrating kindness in this situation?” This way, they can self-assess their behavior and choose to correct it, especially if they are a part of designing the values from the start. This algorithm-based competition was designed to allow teams of students to improve their problem-solving and writing skills through mathematical modeling. Mathematics prowess builds step-by-step over time, and if you take the time to reflect on each of these steps forward, you will connect more with the material and be able to question its overall relevance in your life. Material The number of concepts that can be covered in a covered problem-based curriculum, due to the time required, may be less than what can be covered in a broadcast curriculum. As the teacher, you can guide them on their journey of discovery and help give them the confidence and skills to attack any problem that is presented. This is required in business, academia, and life. The guard that always tells the truth would lie because that is what the other guard would do. According to the first version of Viviani's theorem, this quantity remains constant for regular polygons; hence, it would be constant for equiangular polygons. The first chapter provides a comprehensive introduction to number theory and its mathematical structures. What matters first is to feel that something is wrong (to become aware of it). One of the angles of a regular n-gon is 135°. See Pigeonhole principle Discovery: challenge and, 21; engagement from, 18–19; from play, 12; risk and, 38–41; speed and, 47–48; teachers and, 14 Discrete math, 2–3, 69–70 Dissection time, 239–243 Diversity, 111–112 Divisibility, 86 Doctors, 22 Drive (Pink), 47 Dry erase boards, 98–99 Dudeney, H. Then, by the pigeonhole principle, there is a square with at least three marked points. I am thinking of a number. The product of all natural numbers up to and including the given number – for example, 4! = 4 × 3 × 2 × 1 = 24. The next number would be 728. Try different approaches. The other problems that I love are problems that incorporate lots of different techniques and skills in the same problem. Your instructor and peers are able to help if you get stuck and the You can work to master the material and try your best. Show your vulnerability with problem-solving while at the same time your interest in learning with your child. The triangle has base 20 and altitude 12, so its area is (20 × 12)/2 = 120. The first applications are nice demonstrations that a principle that looks so trivial is actually a subtle tool in proving results. So after an odd number of moves (63) it will be on a square of different color while the opposite corner squares have the same color. Undergraduate research. Draw similar triangles (triangles with at least two equal angles). The third column contains triangular numbers. That is why setting the tone for a kind and helpful environment is so critical. 10. As a teacher, you can have a new job every year as a new crop of students migrates through your class – every year, it is a new job. Renaissance astrologers equated them with planets. LEARNING OBJECTIVES This unit helps develop the counting skills of students. Prove that there are infinitely many Pythagorean triangles, two of whose sides are consecutive integers. American Mathematics Competitions (AMC). Students will have to figure out where they are in terms of these two qualities. They're doing everything right. Thus there are exactly six zeros at the end of 25!. Thinking through these problems primes the brain for mathematical reasoning. We marked all vertices of squares of the table lying at the left and lower edge of the table except for the lower left corner. Clearly, the diagonal sections ACC'A′ and BDD'B′ are also rectangles. How can she measure the length of this diagonal with a ruler? Solutions Because the first two displays use the same number of pieces, we have m(n + 2) − 82 = (m − 2)n − 42, yielding m = 24 − n. 8T2 + 1 = 8 × 3 + 1 = 25 = 55 3. Example 6 Find the largest number of points that can be marked by the vertices of the 20 × 19 table in such a way that no three marked points lie in the vertices of any right-angled triangle. Let P be the intersection point of lines AG and CD. This is a chemistry competition for high school students whose purpose is to inspire excellence in chemistry. If you are in an environment of negative people, it is easy to have a negative version of yourself come to the surface. How have your teaching methods evolved over time, and why? Also, since they aren't responsible for providing solutions or knowing how to solve the problem, it takes away a lot of the pressure they may feel and circumvent math phobia if they have it. Tap the community to see who can help facilitate the club and provide meaningful contributions. J, F, M, A, M, J, ... 11. Hence, the two boxes contain the same amount of chocolate. If you are interested in learning something new, your child will be as well. Collaborative problem-based learning holds the key to unlocking abilities and mathematical growth. Solution: The answer is 56. When a student works at home to write one or two questions to ask in class, they have more confidence and have thought more deeply about the material. In the late 1600s, Isaac Newton wrote a letter to fellow English scientist Robert Hooke, in which he famously said, “If I have seen further, it is by standing on the shoulders of giants.” Newton recognized that his major breakthroughs were only possible because of the groundwork laid by others, and many times, the world would not have been ready for his ideas if they had happened any earlier. The answer is the Math Olympiad Program (MOP), an intensive summer training camp that invites top-scoring students of the USAMO to train together, and from that group, six students are chosen to represent the United States for the IMO. Example 6 Find formulas for the sequences with the following first five terms: a. Two lines that run side by side and never cross or intersect. Even this simple experience can be a wonderful problem-solving opportunity. What qualities do you bring to the classroom? The problem to solve. So a pentagonal number plus a triangular number minus two times a square number is zero. Guiding your child to ask good questions helps: Emphasize the process over outcomes. Helping your students be decisive when required is a leadership quality that propels a group forward if done with kindness and understanding. 4. A classic interview question when applying for a job is, “Where do you see yourself in the next five years?” The purpose of the question is to test the forward thinking of the applicant. Kathy, Alina, and Titu are visionaries in the field of math education, and their book has sparked new inspiration for strategies that I am eager to utilize in my own math classroom.” Hannah Keener “Awesome Math emphasizes the importance of collaborative problem solving in a classroom setting, featuring interesting and carefully chosen concepts and problems that can be used in a regular classroom and enrichment academic mathematics programs such as math circles or summer camps.” Zvezdelina Stankova, Teaching Professor of Mathematics at University of California at Berkeley “This inclusive book speaks in voices of the many. It gives new meaning to the phrase ‘the journey is the reward.’ The biggest danger? These topics include combinatorial arguments and identities, generating functions, graph theory, recursive relations, sums and products, probability, number theory, polynomials, functional equations, complex numbers in geometry, algorithmic proofs, combinatorial and advanced geometry, functional equations, and classical inequalities.Olympiad-style exams consist of several challenging essay problems. On the following picture, lines AX and CY are parallel, ∠XAB = 40° and ∠YCB = 60°. Is your strategy, solution, and process effective and/or efficient? That is impossible. James Joyce Intellectual risks are what stretch us to be better – leaving the comfort zone of our front door and embarking on new quests. All that is requested of them is to help their child ask relevant questions. Thus, F = 2 and S = 3.3 Squaring Numbers: Mental Mathematics OVERVIEW Have you heard of Dr. Arthur Benjamin, the “mathemagician” of our times? Even if the curriculum can't take the place of the requirements dictated by the state or country and/or the students themselves don't have the time in their busy schedules to truly dive into a deep problem-solving approach, there are still mini-units available in the next section of this book where families can be exposed to the joys of problem solving, and hopefully that seed will have the time to bloom later on when external pressures and lack of time aren't so prevalent. This book is mainly aimed at this goal and will help teachers and students improve their logical thinking, making them more independent learners and scholars.” Dr. Krassimir Penev, Bergen County Academies Awesome Math Teaching Mathematics with Problem-Based Learning TITU ANDREESCU KATHY CORDEIRO ALINA ANDREESCU Copyright © 2020 by John Wiley & Sons, Inc. First, note that using diagonals, a polygon can be cut into finitely many triangles. Trapezoid: The area is equal to the product of the height by the half sum of the bases. Any integer not divisible by 2. Here is a simple example that sums to 15 with all the consecutive numbers 1−9. The first seven rows of Pascal's triangle are written with combinatorial notation. I believe it is the teachers' role to pave a good path for any direction the student would like to take to deepen their exploration of a topic and that it is the students' duty to explore as many of those paths as possible through the problems given by the teacher and others. Moreover, Therefore, we must simplify the following: A Wilson prime is a prime number p such that p2 divides (p − 1)! + 1. At first, it is equal to The above expression is equal to ½ × BC(PD + PF + PE). There are numerous competitions with various styles, structures, and goals, however, the one common thread is that they are created by mathematicians who want to excite and engage students in the joys of solving interesting problems. Since GE is both the altitude and median of triangle ABG, this triangle is therefore isosceles. Will I be able to present the solution to the class? On the other hand, I have observed that when the students are in charge of such discussions, and the groups of participants are too big, then the effectiveness of the discussion is very low. See specific topics Purple Comet! Math Meet, 80–81, 104 Purpose: with competition, 50–52; decisiveness and, 110; mastery and, 47; research on, 47 Puzzles: game theory and, 18; for intuition, 163; in suggestion box, 19 Pythagorean Theorem, 32–33, 77, 213–219, 239–240 Pythagoreanism, 200, 204 Q Quadrants, of success, 52–56 Quadrilaterals, 90 Questions: in Cornell method, 100–101; lesson-specific, 97–98; for mini-units, 147–148; for teachers, 77 R Range: curiosity from, 5, 8; exploration and, 9; rigor and, 11 Ratios, 191 Reasoning, 33–34 Receiving (listening), 108 Reciprocals, 255–258 Recognition systems, 108 Rectangles, 89, 213–219, 240 Reflective thinking, 14, 147–148; post reflections, 144–146; resources for, 99–101 Regular polygons, 168 Relativity, 147–148 Relentless curiosity, 77–78 Relevance: in curriculum, 18–21, 62; in logic problems, 19; mathematical, 14–18; in PBL, 77; for students, 13–14 Research: case studies on, 8; in college, 126–127; on creativity, 77; on mastery, 53; on problem solving, 2; on purpose, 47; on STEM, 103– 104 Resilience, 10–12 Resources: for active learning environments, 91–97; classroom environment and, 79–81; knowledge banks, 81–91; mistakes as, 97– 99; for reflective thinking, 99–101; for talent, 99 Retention: in college, 71; Cornell method for, 99–101; for parents, 68; of students, 62 Revisions, 147–148 Rewards: from collaboration, 51; for innovation, 42; for PBL, 5–12 Rhombi, 89, 218 Right angles, 177 Right triangles, 90 Rigor, 9–11 Risk, 1, 38–41 Robinson, Ken, 77 Roleplaying, 49 Roles, for teachers, 108 Roman numeral problems, 148–150 Rose, L. Students who cannot afford these extra enrichment opportunities are left behind. In mathematics, there seems to be a reticence to hear the symphony for fear that it will be too much, too soon, and by limiting the range, we limit curiosity and growth. The ideal would be to have all six characteristics, but as we've stated before, ideals are not always reached. We obtain the solutions (7,733,3), (733,7,3) and cyclic permutations. As a child (and it still holds true), this student loved to create imaginary worlds and lived a lot in his head, which is easy to understand, because it was a fascinating place with so much going on. When running the Metroplex Math Circle, the female participants would always sit in the back of the auditorium and work together. Then ∠XAB = ∠ABD and ∠CBD = ∠DCY. Further, the memories and relationships are so strong that many of the residential and teaching assistants are former campers who return to the program so they can continue to be a part of this kind, engaging environment. The same is true with mathematics, everyone can improve and get better by understanding their particular talents and then determining what areas are the best fit. (This is a problem with a beautiful and unexpected idea in the proof.) Example 3. Mastery Through Inquiry As stated earlier, when you shift the focus from outcomes (grades, rank, solutions) to the journey (thought process, multiple approaches, creativity), then students learn to work toward mastery of a topic and not GPA or other false metrics of ability or worth. Take three squares of dimensions 2 × 2, 3 × 3, and 6 × 6. An n-sided polygon has two interior angles of sizes 94° and 51°. Mathematics education can be just as playful and allow students to compete by solving meaningful problems while working as a team, but that means the stakes need to change, and instead of teachers as judges, saying a student's individual work is good or bad, is missing steps, is B work and not A work, they need to shift into being coaches who guide their students to being the best versions of themselves. Lecture time is very active with constant questions and doing my best to create an environment in which students feel safe to speak up and make mistakes. Thus, 115 is 161 051. Curiosity, vulnerability, courage, and practice will ensure that you are on the right path to offer the most to your students and reap the benefits of a collaborative and supportive class of kids. See also Learning environment Equiangular polygons, 168–170 Equilateral polygons, 168–170, 236 Equilateral triangles, 241 Ergodic theory, 229, 233 Euler, Leonhard, 160, 216 Euler bricks, 213, 216–219 Evaluation, xvii Even numbers, 185 Exercises, 9 Expectations, 27–28, 44 Experience: in community outreach, 3; inexperience, 63, 69; judgments and, 113; with PBL, 128–129, 132–134, 136–137; for teachers, 125–126, 136 Exploration, 9 F Factorials, 85, 191–197 Fagnano, Giulio Carlo de' Toschi di, 145 Fear, Uncertainty, and Doubt (F.U.D.), 8 Feedback: curiosity and, 50; in PBL, 56; from teachers, 48 Fermat, Pierre de, 144–145 Fermat primes, 253 Feynman, Richard, 21–22 Fibonacci numbers, 18, 84–85, 225, 247 Field, Rachel, 54 Financial capital, 113–114 Finite sets, 157–159, 239 Fixed mindset approaches, 27–28 Flexibility, 29, 73 Flipped classrooms, 37–38, 58, 101 Flow, 31 Focus, 63 Forestry, 165 Fractions, 81–82, 255–258 Franklin, Benjamin, 160 F.U.D. See Fear, Uncertainty, and Doubt Fun. The distinction between a truth versus power approach is an important one. It took a year for anyone to achieve a perfect score in her class on a test once she made the change. What areas were difficult to understand? You provide the students with more control over a situation because they can always control themselves. The sum of nine odd consecutive numbers is 2007. Do they ask for the story behind the problem or more facts about it? When asked, “What would you do to help a student who is stuck on a problem?” his answer was, “I would never be so cruel as to spoil the solution for them. This unit is accessible to middle school and high school students. Understand the value of mistakes. What don't you know about the fear? The volume of a sphere of radius r is V = (4/3)πr3. Let ∠CPA = α, ∠CPG = ∠APD = β. If discussing a non-negative length x, then x = √x2. As Dr. Andreescu has noted in his vast coaching experience: It's like you are in a tennis match with the student; they may be faster and more agile, but you know where the ball is going. Taking into account the carryover, looking at the tens digit we have C + 1 = B (we cannot have C + 1 = B + 10 because that would force B = 0, a contradiction). Easy In a magic square, the sum of the three entries in each row, column, or diagonal has the same value. The least is obtained by writing the smallest valued numerals before the larger ones: MCDXLIV, which represents 1444. This means that ∠GCD = ∠GDC = 70°. Each of the success quadrants are interdependent; therefore, timing and luck require the individual to be a part of community (people) so that feedback and guidance can be available. Having a network of educators who could share ideas, feedback, material, and support opened my eyes to the various active learning options and emboldened me to try them in my own classes. MATHEMATICS COMPETITIONS Purple Comet! Math Meet. Observe that a1, a2, ..., ak are summands (terms) in a representation of a nice number, then 1/2a1 + 1/2a2 + ... + 1/2ak = 1/2. They can get excited because they will partially read, think they know what the problem is asking, and will impetuously start solving. The top four individual competitors from each State Competition advance to the National Competition, which takes place in May. Can I describe the solution clearly? When the war ended, and the air strips were dismantled, the tribes still wanted to continue to receive the material wealth of the cargo. Encouraging conversations between students during class is a cornerstone of my approach to teaching. Consider the set A = {a|a is positive integer less than 2009 and 7|a}. Wait until college, and then what you've been learning in high school will be useful. PROFESSIONAL LEARNING COMMUNITY (PLC) Having a community to offer you support as you create a PBL classroom has multiple benefits. (Example: 12 and 25 are co-prime. The social aspect of collaborative problem solving is immeasurably beneficial for developing young minds. The curriculum included in Section III of this book is intended to mitigate these cons by providing the lecture notes and scaffolded problems. A fraction with a numerator of 1. This 15question, 3-hour exam requires that each answer be an integer number from 0 to 999. Personally, I have gravitated toward utilizing group work in whatever capacity the class schedule and physical space will allow. Marilyn Monroe was born in MCMXXVI, and died in MCMLXII. He has high energy and a large heart. Mathematical Relevance: The Math Circle Example Mathematical relevance is responsible for much of the success of math circles across the globe. Therefore, 90° − θ = 72°. Show relentless curiosity. Solution We will solve a slightly more general problem, replacing 20 by a, 30 by b, and 60 by c. Sums of reciprocals of positive integers have been used to express other rational numbers since the ancient times. 20, 22, 24, 26. See Teachers Collaboration, xviii; challenge and, 42; in classrooms, 112; in community outreach, 6; competition and, 73; creativity from, 36–37, 79; in environment, 15; groups for, 10, 50; identity capital in, 113–114; innovation and, 76–77; leadership in, 108–112; learning from, 92; mathematical games for, 114–116; in PBL, 48, 55, 58, 112, 123–125, 130, 135; rewards from, 51; rotation and, 96; schedules for, 118–119; skills from, 3, 7; for students, 107–108; by teachers, 29; trade in, 113 College, 71, 126–127 Combinations, 248–249 Combinatorial set theory, 228 Combinatorics, 7, 191, 245 Communication, 98, 109 Community: competition and, 117–118; map of engagement for, 120–121; math circles for, 119; math clubs for, 118–119; parents in, 120– 121; PBL and, 103; PLC, 17, 119–120 Community outreach: challenge in, 92–93; collaboration in, 6; experience in, 3; PBL in, 19, 92; teachers in, 40, 54, 112 Competition: collaboration and, 73; community and, 117–118; engagement and, 3; for groups, 15; Mandelbrot Competition, 105; in mathematics, 103–106; in PBL, 106–107; purpose with, 50–52; STEM and, 16; timing in, 78 Complex problems, 24 Computational linguistics, 7 Concentric circles, 219 Confidence, 98, 109 Congruent triangles, 168, 177 Connection, 42 Conrad Challenge, 107 Consecutive numbers, 185–189, 227 Continuous challenge, 60, 63, 68 Control, 65 Conversations, 92–93 Coopertition, 15, 50, 103 Coping, 64 Co-prime integers, 227–229 Core values, 43–44 Cornell method, 99–101 Counting, 20 Creativity: active, 54; from collaboration, 36–37, 79; flexibility and, 29, 73; innovation and, 109–110; introspection and, 31–32; play and, 96; research on, 77; from resilience, 10–12 Crisan, Vlad, 131–132 Critical thinking, xviii; pedagogy for, xix; about Pythagorean Theorem, 77; rigor for, 9–10; scale in, 20; understanding and, 24 Cryptarithmetic, 151–155 Cubes, 84 Curiosity: engagement and, 49; feedback and, 50; practice and, 99; from range, 5, 8; as relentless, 77–78 Curriculum: for education, xviii; incomplete, 61; lectures in, 38; for PBL, 14–15, 39–40, 52–53; relevance in, 18–21, 62; risk in, 1. Correct solutions often require deep analysis and careful argument. Vertex (Vertices pl.). In the Monomyth, the threshold can be as simple as going into a dark and mysterious cave where you have adventures and come out the other side. Be excited to share and go beyond the lesson. For Kathy Cordeiro, innovation, problem solving, and team collaboration have been the leading constants throughout her varied career. This is a common type of problem and can help build problem-solving skills for each student. I do this much more today than I did years ago. As highlighted in Chapter 4, in Daniel Pink's book, Drive, when individuals have autonomy (control over the process), mastery (skills to succeed), and purpose (meaningful work), they will be internally motivated and a leader can help nurture these characteristics. This was the kid who would ask why people were crying, what happens when you die, what is the last connection you would want to make with someone. L. We need to find the space diagonals length AC′ = g in terms of a, b, and c. (Be sure to check out the mini-unit on magic squares to delve deeper into this topic!) Be authentic, honest, and positive. Rectangular Boxes Example 1 What is the length of a solid diagonal of a 20 × 30 × 60 rectangular box? Dean Schlicter Take time to explore other areas of mathematics such as logic, puzzles, game theory, Fibonacci numbers, magic squares, set theory, math and the Rubik's cube, Conway's game of life, math card tricks, knot theory, infinity, KenKen, geometric constructions, and on and on and on! There are so many rabbit holes to go down, and that is the joy of the math circle – taking the time to go down the hole with a guide who is just as curious and excited by the discovery as their students! Curriculum Relevance Learning the story behind discoveries also brings mathematics to life and makes it relevant. F, 4, E, S, 9, S, E, 5, E, ... 15. OTHER PROBLEM-SOLVING-BASED CONTESTS The following contests also have a problem-solving-based approach and offer further opportunities for students who enjoy critical thinking, logic, collaboration, and an engaging and fun community: North American Computational Linguistics Olympiad (NACLO). A question that comes up all the time from parents is “How do I limit my child's time playing video games?” Time is such an arbitrary way to affect choices. These problems can be solved with very little knowledge of geometric formulas, and they can be visualized with physical materials. The IOI is the most prestigious international computing contest at the high school level. They need to see the strengths of the team members and hold them to task for those capabilities, while having compassion for each team member. We have a large pile of identical 20 gram 1 × 1 square pieces. Flow is when you are process versus results oriented. There are no limitations? The properties taught in this lesson were proved by F. This is slightly different from perfectionism in that it's not fear of mistakes as much as holding yourself to high personal expectations of performance, and if you don't think you can meet them, you won't try. Dr. Andreescu was also the leader of the US IMO team the year Dr. Loh competed (1999) and won a silver medal. Accountability can help provide the impetus to get the team going. During that time, I learned mathematics from the modern standpoint, e.g., Euclidean geometry taught based on the Hilbert's axioms, trigonometry, basics of linear algebra, plane analytical geometry, and the medley of topics known as algebra, but with the relevant theory thoroughly discussed. Communication is a two-way street. It's because of the student who owns a bird, a cat, and a dog who was subtracted three times instead of twice, so he needs to be added back in to equal 31. Find the greatest of these nine numbers. Is there another way to think about this problem? If n = 2k + 1 ≥ 1, then (2k + 1)(m + k) = p. Visualize the 64 squares of the chessboard, its rows, columns, and diagonals. Example 1 Prove that a square can be dissected into n squares for all n > 6. This is where grace comes into play and giving yourself permission to try new things, fail, get up, iterate, and try again in a different way. Just like in gaming, when you get online with friends to play, it creates more excitement and connection with the material when you can share the experience. Or something else entirely creates a roadblock. Nevertheless, I do pay a lot of attention to and keep track of the feedback forms that I write for them. If 2 students have a bird and a cat, 4 students have a bird and a dog, 6 students have a cat and a dog, and 1 student has all three pets, how many students

League (ARML). From y + 6 + 5 = x + 4 + 5 it follows that y = x − 2. Then, you can better help your child understand the question they need to ask. This view creates a love of learning and a resilience that is essential for great accomplishment.”5 Agile development. Connect. The analysis of chess can be extremely complicated due to many possible options at each move. Vision and perspective. From the condition we know that 1/a1 + 1/a2 + … + 1/ak = 1. Factorials are simple functions with many applications in the study of non-negative integers. As an example, I would like to discuss a problem I love to share with my undergraduate students in the frames of a course on extracurricular math activities. We shall use two important mathematical facts: 1. When tested later, student's retention of topics lasted longer with a problem-based approach. He was a literature professor whose research was in comparative mythology and comparative religion. 17. Research the story of algebra (why, how, and by whom was it created). See also Groups Styles, 30, 45–46 Subsets, 227 Success, 52–56 Suggestion box: ancient number systems in, 9; for anxiety, 64; for AwesomeMath Enrichment programs, 45, 49; biographies in, 18; game theory in, 11; logic problems in, 18; movement in, 21; for PBL, 7; puzzles in, 19; for students, 163; for teachers, 51 Supplementary angles, 177 Support, 69 Suspense, 50 Systems: for approaches, 35–36; for problem-based curriculum, 57; for recognition, 108; for understanding, 45–46 T Talent, 52–53; innate, 99; nurturing of, 99; resources for, 99 Teacher inspiration: AwesomeMath Enrichment programs and, 121– 122; from Crisan, 131–132; from Herzig, 124–125; from Kisacanin, 134–135; from Langarica, 125–128; from Mushkarov, 136–144; from Newberry, 122–124; from Pompe, 136; post reflections after, 144–146; from Turcas, 132–134; from Yotov, 128–131 Teachers: assistants, 80; authenticity by, 94; case studies on, 58; challenge from, 10; chunking by, 80–81; classroom management for, 93–96; collaboration by, 29; in community outreach, 40, 54, 112; confidence for, 109; coping and, 64; discovery and, 14; doctors and, 22; efficiency for, 61; encouragement from, 49; engagement by, xviii; experience for, 125–126, 136; feedback from, 48; gains for, 59; guidance by, 56; The Housekeeper and the Professor, 15; humility for, 109; idealism for, 76; inexperience of, 63, 69; judgments by, 80; learning environment and, 33–35; methods for, 123–125, 127, 130, 134; pains for, 60–61; parents and, 70; PBL for, 73–74; planning by, 44–46; PLCs for, 17; problem-based curriculum, 58–61; questions for, 77; roles for, 108; students and, 2, 92, 194; styles for, 45–46; suggestion box for, 51; value propositions for, 59. If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. Autonomy to Solve Your Problems Having the power to work on problems in your own way, trying different paths and approaches, coupled with kind guidance from the teacher, provides students the autonomy to learn at their own speed and by choosing their own paths. So How Do You Create a Space Where Students Can Be Vulnerable and Ask for Help? This is the Fibonacci sequence (each term is obtained by adding the two previous terms). When the 7 minute timer runs out, turn the 4 minute time over and it will run for the required extra 3 minutes. Talking about the world? Obviously, the applicant had the necessary math skills for the job, so the next concern was did he have the right approach when working with students? To see if there is a math circle near you, visit the National Association of Math Circles5 website. Source: AwesomeMath Admission Test A, 2019. This reduces the stigma that can come with fitting a one-size-fits-all model. Let n = a1 + a2 + … + ak. The quick reason is “for their academic future.” Many universities are looking beyond SAT or ACT scores for applicants to science, technology, engineering, mathematics (STEM) programs. Therefore, the sum of those Roman numerals is equal to the 36th triangular number. Explanation: Whatever cave the guard tells you to take, you would take the opposite. Describe where algebra is used in life (why, how, and when it is necessary today). Solution Divide numbers from the set, in pairs (1,10), (2,9), (3,8), (4,7), (5,6). Example 3 How many integers from 1 to 500 are divisible by 3 or by 7? Note that k = 180° − (360°/n) is a positive integer. The next number is the sequence is 16. The teacher may not have the mathematics background necessary to teach problem solving at this level. Standardized tests, standard time to learn concepts, standard material. The list that follows are some tried-and-true techniques, but that doesn't mean you can't innovate your own approach based on the students in your class and/or experience. So the next one would be X. Do they see a pattern? They want to pin that knowledge in the fabric of their creative brain space by connecting it to other ideas, finding the story behind it, or deconstructing/reconstructing the problem. A polygon is called equilateral if all of its sides are equal. Encourage all questions and the process to learn. First, it's important to remember that we all lead in different ways based on who we are and what we want to achieve. Thus, 7 is not a Wilson prime. Any time you can guide a student to relate, reflect, and revise, it will help them have better retention, think critically, be an independent learner, and, most importantly, grow their understanding and passion for mathematics and problem solving. The group that I graduated with was a group who wanted to be challenged academically. Game Players Type 24 Puzzle Buzz 2+ Chess 2 Chess puzzles Chomp Equate Versus (non- A few minutes None zero-sum) Versus (zero- 30–60 Chess set sum) Puzzle 2 2–4 or teams Time Materials commitment Optional deck of cards Chess set Versus (zero- A few minutes 40 or more sum) markers, 1 a different color Versus (zero- 30–60 min Equate game sum) Game of life Simulation Geometric construction Puzzle Hive Logic puzzles Mancala 2 2–4 Computer, optional graph paper Straightedge and compass Versus (zero- 15–30 sum) Hive set Puzzle None Versus (zero- 15–30 sum) Mancala set Mastermind 2 Puzzle Prime Climb 2–4 10 min per player Nim Versus (zero- A few minutes 10 or more sum) markers 2 Northcott's 2 game Reversi 2 Set 1+ A few minutes Optional Mastermind set 15–30 Prime Climb game Versus (zero- A few minutes Graph paper, sum) 3–8 markers in each of two colors Versus (zero- 30–60 Reversi/Othello sum) set Puzzle/Versus 5–30 “Set” cards ThinkFun puzzles Towers of Hanoi (non-zerosum) Puzzle Puzzle ThinkFun puzzles or equivalent Stackable discs or markers of increasing size Community COMPETITION COMMUNITY Finding your tribe is important for all of us. Besides the question of what such objects are or could be, is the problem of how we can make sense of such spaces. See below: CHAPTER 19 Pascal's Triangle Learning Objective Pascal's triangle is a single mathematical structure that is incredibly dense with important subfigures and identities. You've solved similar problems before. In this section, you will learn about the competitions available for students of various ages, skills, and interests and can choose what works best in your classroom. Interpersonal Conflict resolution, problem solving, decision making, skills and leadership skills all flourish in a team-based collaborative environment. Solution We calculate the area of the triangle ABC in two different ways. The answer is 90. Solution Roman numeral values are as follows: D = 500, C = 100, L = 50, X = 10, V = 5, I = 1. They didn't grow up with broadcast television where you need to sit in front of your TV at the prescribed time to watch your favorite show. These teachers prepared students for math competitions, including the IMO. The following diagram shows a visual way of imagining a grid. Is it a better design for me the individual? The goal of coaching is to allow each student to have the ability to say Veni, Vidi, Vici – in other words, I know how I learn, I know how to solve today's problems, and I have the confidence to conquer other problems in my path. Many times, I give the students 5–10 minutes individually to work to formulate a solution to a problem. See Teachers Efficacy: in communication, 98; in learning, 92–93, 123, 127, 129–136; in PBL, 124; of strategies, 23 Efficiency, 33, 61 Ego, 27–29 Einstein, Albert, 18 Embedding proofs, 237–238 Empathy, 64 Encouragement: from parents, 121; for students, 79–80; from teachers, 49 The End of Average (Rose), 36 Engagement: active, 62, 68; from challenge, 15, 80; competition and, 3; with complex problems, 24; curiosity and, 49; from discovery, 18– 19; goals and, 53; with logic problems, 38; map of, 70–71, 120–121; with parents, 37; from play, 8, 17; with process, 121; strategies for, 54– 55; with students, 75–76; by teachers, xviii Enrichment centers, 25 Enrichment schools, 119 Environment: for classrooms, 79–81; collaboration in, 15. It follows that (p − 6)(q − 6) = 727, and since 727 is a prime, we get p − 6 = 1 and q − 6 = 727 or p − 6 = 727 and q − 6 = 1. Find all n for which there are n consecutive integers whose sum is a prime number. Busy Problem-based learning requires slow and deep schedules thinking for which today's overscheduled student doesn't have the time. Finding relationships between angles is a good exercise for interpreting geometric diagrams and extracting meaningful information. Let's say that Steve is playing a video game with a friend and loses a boss battle. Now, as an adult, his passions are pure mathematics, theoretical computer science, and creating video games. Finding its minimum gives us a genuine solution (Figure 7.10). Therefore, since 200 = 8 × 25 = 40 × 5 are the only possible decompositions of 200 in two factors with different parities, we find that, either 2a + k = 25, k + 1 = 8 or 2a + k = 40, k + 1 = 5. Problem solving brings real-world problems to their front step and shows them how imperative it is to have mathematical understanding. Students are clever and they know when they are given the control to grow versus told how to grow. With problem-based learning, you aren't lockstepping the kids together at the same rate; instead, you are letting them develop in their own way and at their own speed. Solution The answer is 14. The point at which two line segments, lines, or rays meet to form an angle. Analyze the variables that may affect the system. It is highly collaborative because when you engage in the trade of ideas, everyone improves. Math circles don't have to be held on college campuses. This is even more apparent in math courses. You can ask one question; what would it be? Was there a topic they liked more than others? Therefore, the study of Roman numerals helps people to better analyze and understand the workings of the world. Triangular numbers have a wide variety of relations to other polygonal numbers. It is obvious that the sum (or difference) of any two distinct numbers from this set is never a multiple of 2018. Creating effective collaboration teams, while extremely beneficial for PBL, also provides useful classroom benefits such as: Inclusion and diversity Importance of trade Identity capital Mathematical games Dr. Alicia Prieto Langarica How Have Your Teaching Methods Evolved Over Time, and Why? Most times, this means I have a very hands-off approach to research, letting students decide the direction our problem should take us, what methods we should explore and know that mistakes will be made and that is actually a great thing. He created a 16 × 16 square that astounded others. The front right then had a choice of 3 letters out of 25, the rear left paw had a choice of 2 out of 24 letters, and the rear right had only one favorable choice out of 23, for the final probability of Usefulness Learning needs to be useful as well as relevant and authentic. PROBLEMS There are many interesting problems with toothpicks. See American Mathematics Competitions American Invitational Mathematics Exam (AIME), 103–104, 106 American Mathematics Competitions (AMC), 103–104, 105 American Regions Mathematics League (ARML), 106 Ancient number systems, 9 Angle bisectors, 91 Angles: definitions of, 177; parallel lines and, 177–183; in polygons, 178–179; in triangles, 180–183 Anxiety, 64 Apothem polygons, 237 Approaches: bottoms-up, 36–37; core values in, 43–44; discovery in, 38–41; fixed mindset, 27–28; flipped classrooms, 37–38; for individuals, 36–37; mission statements for, 41–43; sage on the stage approach for, 20; systems for, 35–36; in testing, 45; top-down, 36–37 Area, 91, 165, 172–173, 235–236, 239 Arithmetic progressions, 221, 223 ARML. The essence of math is not to make simple things complicated, but to make complicated things simple. My first-year high school teacher obviously knew how to motivate students to work and do their best! The second similar recognition came when my algebra teacher invited me to do a project in math when I was in my third year at high school. The last digit will be 1, 3, 5, 7, or 9. Then, writing that the sum of the entries of the first row is equal to the sum of the entries of the second column, we get (x − 2) + z + 3 = 4 + 6 + z, implying x = 9. If you had a question at the Mozart table, you would ask someone at your table and/or someone at the Bach table, and students at the Bach table could confer with each other or with Kathy as their coach. Did you ask for help from peers or the teacher? a2 ≥ 0 for all real numbers a 2. 13. The number of possible teams is 2. Encourage students when they show initiative, innovate, and leave their comfort zones. On the other hand, let O be the center of the polygon. As a leader, you need to make sure the team is on task and understands what is required, so this means a certain level of “send” has to happen. Every row, column, and diagonal will have the same sum. Teaching the next generation of mathematical thinkers requires introspection, analysis, understanding, and a plan. PROBLEMS We present two examples of how we can use these identities for quick “mental” squaring of numbers (without a calculator). If the angles at a base of a trapezoid are equal, then it is isosceles. CHAPTER 18 Dissection Time Learning Objectives Students can find the area and perimeter of a geometric figure, dissect it into pieces, and describe the attributes of the resulting shape. I also like to ask reflection questions on exams. large square. You can learn more about Viviani's theorem in Chapter 17. Class size Classes over 20 students create a chaotic environment where it can be difficult to focus if the teacher cannot manage the class. Set up scenarios where the students can role play what they think a kind and collaborative group would look like and build from there. Working in groups also provides the accountability to participate, helping to ensure that students actually watch the lecture, so they can add value to their group. Understanding the world better? If they are fearful of being wrong, it completely defeats the purpose. Also, we know that the sum of the angles in an n-sided polygon is equal to (n − 2) × 180°. In our case (a = 20, b = 30, and c = 60), . Richard Feynman Dr. Feynman was stating that the missing piece was the integrity, truth, and honesty that you are working to find a solution to a meaningful problem instead of following a bias or mimicry or glory. Finite. The smallest Euler brick, discovered by Paul Halcke in 1719, has edges (a,b,c) = (44,117,240) and face diagonals (d,e,f) = (125,244,267).

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