Function of a graph examples

Representation of a function as the set of pairs (x, f(x)) For graphical representation, see Plot (graphics). For the combinatorial structure, see Graph (discrete mathematics). For the graph-theoretic representation of a function from a set to itself, see Functional graph. This article needs additional citations for verification. Please help improve this article by adding citations to reliable sources. Unsourced material may be challenged and removed.Find sources: "Graph of a function" – news · newspapers · books · scholar · JSTOR (August 2014) (Learn how and when to remove this template message) Graph of the function f(x) = x3 − 9x In mathematics, the graph of a function f {\displaystyle f} is the set of ordered pairs (x , y) {\displaystyle (x,y)} , where f (x) = y {\displaystyle f(x)=y} . In the common case where x {\displaystyle x} and f (x) {\displaystyle f(x)} are real numbers, these pairs are Cartesian coordinates of points in two-dimensional space and thus form a subset of this plane. In the case of functions of two variables, that is functions whose domain consists of pairs (x, y), the graph usually refers to the set of ordered triples (x , y , z) {\displaystyle (x,y,z)} where f (x , y) = z {\displaystyle f(x,y)=z} , instead of the pairs ((x , y) , z) {\displaystyle ((x,y),z)} as in the definition above. This set is a subset of three-dimensional space; for a continuous real-valued function of two real variables, it is a surface. A graph of a function is a special case of a relation. In science, engineering, technology, finance, and other areas, graphs are tools used for many purposes. In the simplest case one variable is plotted as a function of another, typically using rectangular axes; see Plot (graphics) for details. In the modern foundations of mathematics, and, typically, in set theory, a function is actually equal to its graph.[1] However, it is often useful to see functions as mappings,[2] which consist not only of the relation between input and output, but also which set is the domain, and which set is the codomain. For example, to say that a function is onto (surjective) or not the codomain should be taken into account. The graph of a function on its own doesn't determine the codomain. It is common[3] to use both terms function and graph of a function since even if considered the same object, they indicate viewing it from a different perspective. Graph of the function f(x) = x4 − 4x over the interval [−2,+3]. Also shown are the two real roots and the local minimum that are in the interval. Definition Given a mapping f : X → Y {\displaystyle f:X\to Y} , in other words a function f {\displaystyle f} together with its domain X {\displaystyle X} and codomain Y {\displaystyle Y} , the graph of the mapping is[4] the set G (f) = { (x , f (x)) | x ∈ X } {\displaystyle G(f)=\{(x,f(x))\mid x\in X\}} , which is a subset of X × Y {\displaystyle X\times Y} . In the abstract definition of a function, G (f) {\displaystyle G(f)} is actually equal to f {\displaystyle f} . One can observe that, if, f : R n → R m {\displaystyle f:\mathbb {R} ^{n}\to \mathbb {R} ^{m}} , then the graph G (f) {\displaystyle G(f)} is a subset of R n + m {\displaystyle \mathbb {R} ^{n+m}} (strictly speaking it is R n × R m {\displaystyle \mathbb {R} ^{n}\times \mathbb {R} ^{m}} , but one can embed it with the natural isomorphism). Examples Functions of one variable Graph of the function f(x, y) = sin(x2) · cos(y2). The graph of the function f : { 1 , 2 , 3 } → { a , b , c , d } {\displaystyle f:\{1,2,3\}\to \{a,b,c,d\}} defined by f (x) = { a , if x = 1 , d , if x = 2 , c , if x = 3 , {\displaystyle f(x)={\begin{cases}a,&{\text{if }}x=1,\\d,&{\text{if }}x=2,\\c,&{\text{if }}x=3,\end{cases}}} is the subset of the set { 1 , 2 , 3 } × { a , b , c , d } {\displaystyle \{1,2,3\}\times \{a,b,c,d\}} G (f) = { (1 , a) , (2 , d) , (3 , c) } . {\displaystyle G(f)=\{(1,a),(2,d),(3,c)\}.\,} From the graph, the domain { 1 , 2 , 3 } {\displaystyle \{1,2,3\}} is recovered as the set of first component of each pair in the graph { 1 , 2 , 3 } = { x : there exists y , such that (x , y) ∈ G (f) } {\displaystyle \{1,2,3\}=\{x:\ {\text{there exists }}y,{\text{ such that }}(x,y)\in G(f)\}} . Similarly, the range can be recovered as { a , c , d } = { y : there exists x , such that (x , y) ∈ G (f) } {\displaystyle \{a,c,d\}=\{y:{\text{there exists }}x,{\text{ such that }}(x,y)\in G(f)\}} . The codomain { a , b , c , d } {\displaystyle \{a,b,c,d\}} , however, cannot be determined from the graph alone. The graph of the cubic polynomial on the real line f (x) = x 3 − 9 x {\displaystyle f(x)=x^{3}-9x\,} is { (x , x 3 − 9 x) : x is a real number } . {\displaystyle \{(x,x^{3}-9x):x{\text{ is a real number}}\}.\,} If this set is plotted on a Cartesian plane, the result is a curve (see figure). Functions of two variables Plot of the graph of f(x, y) = −(cos(x2) + cos(y2))2, also showing its gradient projected on the bottom plane. The graph of the trigonometric function f (x , y) = sin (x 2) cos (y 2) {\displaystyle f(x,y)=\sin(x^{2})\cos(y^{2})\,} is { (x , y , sin (x 2) cos (y 2)) : x and y are real numbers } . {\displaystyle \{(x,y,\sin(x^{2})\cos(y^{2})):x{\text{ and }}y{\text{ are real numbers}}\}.} If this set is plotted on a three dimensional Cartesian coordinate system, the result is a surface (see figure). Oftentimes it is helpful to show with the graph, the gradient of the function and several level curves. The level curves can be mapped on the function surface or can be projected on the bottom plane. The second figure shows such a drawing of the graph of the function: f (x , y) = − (cos (x 2) + cos (y 2)) 2 {\displaystyle f(x,y)=-(\cos(x^{2})+\cos(y^{2}))^{2}\,} Generalizations The graph of a function is contained in a Cartesian product of sets. An X–Y plane is a cartesian product of two lines, called X and Y, while a cylinder is a cartesian product of a line and a circle, whose height, radius, and angle assign precise locations of the points. Fibre bundles are not Cartesian products, but appear to be up close. There is a corresponding notion of a graph on a fibre bundle called a section. See also Asymptote Chart Concave function Convex function Contour plot Critical point Derivative Epigraph Normal to a graph Slope Stationary point Tetraview Vertical translation y-intercept References ^ Charles C Pinter (2014) [1971]. A Book of Set Theory. Dover Publications. p. 49. ISBN 978-0-486-79549-2. ^ T. M. Apostol (1981). Mathematical Analysis. Addison-Wesley. p. 35. ^ P. R. Halmos (1982). A Hilbert Space Problem Book. Springer-Verlag. p. 31. ISBN 0-387-90685-1. ^ D. S. Bridges (1991). Foundations of Real and Abstract Analysis. Springer. p. 285. ISBN 0-387-98239-6. External links Wikimedia Commons has media related to Function plots. Weisstein, Eric W. "Function Graph." From MathWorld—A Wolfram Web Resource. Retrieved from " Not to be confused with the partial application of a function of several variables, by fixing some of them. Function whose actual domain of definition may be smaller than its apparent domain This article includes a list of general references, but it remains largely unverified because it lacks sufficient corresponding inline citations. Please help to improve this article by introducing more precise citations. (August 2014) (Learn how and when to remove this template message) Functionx ↦ f (x) Examples of domains and codomains X {\displaystyle X} → N {\displaystyle \mathbb {N} } , N {\displaystyle \mathbb {N} } → X {\displaystyle X} , N n {\displaystyle \mathbb {N} ^{n}} → X {\displaystyle X} X {\displaystyle X} → Z {\displaystyle \mathbb {Z} } , Z {\displaystyle \mathbb {Z} } → X {\displaystyle X} X {\displaystyle X} → R {\displaystyle \mathbb {R} } , R {\displaystyle \mathbb {R} } → X {\displaystyle X} , R n {\displaystyle \mathbb {R} ^{n}} → X {\displaystyle X} X {\displaystyle X} → C {\displaystyle \mathbb {C} } , C {\displaystyle \mathbb {C} } → X {\displaystyle X} , C n {\displaystyle \mathbb {C} ^{n}} → X {\displaystyle X} Classes/properties Constant Identity Linear Polynomial Rational Algebraic Analytic Smooth Continuous Measurable Injective Surjective Bijective Constructions Restriction Composition λ Inverse Generalizations Partial Multivalued Implicit vte In mathematics, a partial function f from a set X to a set Y is a function from a subset S of X (possibly X itself) to Y. The subset S, that is, the domain of f viewed as a function, is called the domain of definition of f. If S equals X, that is, if f is defined on every element in X, then f is said to be total. More technically, a partial function is a binary relation over two sets that associates every element of the first set to at most one element of the second set; it is thus a functional binary relation. It generalizes the concept of a (total) function by not requiring every element of the first set to be associated to exactly one element of the second set. A partial function is often used when its exact domain of definition is not known or difficult to specify. This is the case in calculus, where, for example, the quotient of two functions is a partial function whose domain of definition cannot contain the zeros of the denominator. For this reason, in calculus, and more generally in mathematical analysis, a partial function is generally called simply a function. In computability theory, a general recursive function is a partial function from the integers to the integers; for many of them no algorithm can exist for deciding whether they are in fact total. When arrow notation is used for functions, a partial function f {\displaystyle f} from X {\displaystyle X} to Y {\displaystyle Y} is sometimes written as f : X ⇀ Y , {\displaystyle f:X\rightharpoonup Y,} f: X ↛ Y, f : X ⇸ Y , {\displaystyle f:X\nrightarrow Y,} or f : X ↪ Y . {\displaystyle f:X\hookrightarrow Y.} However, there is no general convention, and the latter notation is more commonly used for injective functions.[citation needed]. Specifically, for a partial function f : X ⇀ Y , {\displaystyle f:X\rightharpoonup Y,} and any x ∈ X , {\displaystyle x\in X,} one has either: f (x) = y ∈ Y {\displaystyle f(x)=y\in Y} (it is a single element in Y), or f (x) {\displaystyle f(x)} is undefined. For example, if f {\displaystyle f} is the square root function restricted to the integers f : Z → N , {\displaystyle f:\mathbb {Z} \to \mathbb {N} ,} defined by: f (n) = m {\displaystyle f(n)=m} if, and only if, m 2 = n , {\displaystyle m^{2}=n,} m ∈ N , n ∈ Z , {\displaystyle m\in \mathbb {N} ,n\in \mathbb {Z} ,} then f (n) {\displaystyle f(n)} is only defined if n {\displaystyle n} is a perfect square (that is, 0 , 1 , 4 , 9 , 16 , … {\displaystyle 0,1,4,9,16,\ldots }). So f (25) = 5 {\displaystyle f(25)=5} but f (26) {\displaystyle f(26)} is undefined. Basic concepts An example of a partial function that is injective. An example of a function that is not injective. A partial function is said to be injective, surjective, or bijective when the function given by the restriction of the partial function to its domain of definition is injective, surjective, bijective respectively. Because a function is trivially surjective when restricted to its image, the term partial bijection denotes a partial function which is injective.[1] An injective partial function may be inverted to an injective partial function, and a partial function which is both injective and surjective has an injective function as inverse. Furthermore, a function which is injective may be inverted to an injective partial function. The notion of transformation can be generalized to partial functions as well. A partial transformation is a function f : A ⇀ B , {\displaystyle f:A\rightharpoonup B,} where both A {\displaystyle A} and B {\displaystyle B} are subsets of some set X . {\displaystyle X.} [1] Function A function is a binary relation that is functional (also called right-unique) and serial (also called left-total). This is a stronger definition than that of a partial function which only requires the functional property. Function spaces The set of all partial functions f : X ⇀ Y {\displaystyle f:X\rightharpoonup Y} from a set X {\displaystyle X} to a set Y , {\displaystyle Y,} denoted by [X ⇀ Y] , {\displaystyle [X\rightharpoonup Y],} is the union of all functions defined on subsets of X {\displaystyle X} with same codomain Y {\displaystyle Y} : [X ⇀ Y] = ⋃ D ⊆ X [D → Y] , {\displaystyle [X\rightharpoonup Y]=\bigcup _{D\subseteq {X}}[D\to Y],} the latter also written as ⋃ D ⊆ X Y D . {\textstyle \bigcup _{D\subseteq {X}}Y^{D}.} In finite case, its cardinality is | [X ⇀ Y] | = (| Y | + 1) | X | , {\displaystyle |[X\rightharpoonup Y]|=(|Y|+1)^{|X|},} because any partial function can be extended to a function by any fixed value c {\displaystyle c} not contained in Y , {\displaystyle Y,} so that the codomain is Y ∪ { c } , {\displaystyle Y\cup \{c\},} an operation which is injective (unique and invertible by restriction). Discussion and examples The first diagram at the top of the article represents a partial function that is not a function since the element 1 in the left-hand set is not associated with anything in the right-hand set. Whereas, the second diagram represents a function since every element on the left-hand set is associated with exactly one element in the right hand set. Natural logarithm Consider the natural logarithm function mapping the real numbers to themselves. The logarithm of a non-positive real is not a real number, so the natural logarithm function doesn't associate any real number in the codomain with any non-positive real number in the domain. Therefore, the natural logarithm function is not a function when viewed as a function from the reals to themselves, but it is a partial function. If the domain is restricted to only include the positive reals (that is, if the natural logarithm function is viewed as a function from the positive reals to the reals), then the natural logarithm is a function. Subtraction of natural numbers Subtraction of natural numbers (non-negative integers) can be viewed as a partial function: f : N × N ⇀ N {\displaystyle f:\mathbb {N} \times \mathbb {N} \rightharpoonup \mathbb {N} } f (x , y) = x − y . {\displaystyle f(x,y)=x-y.} It is defined only when x ≥ y . {\displaystyle x\geq y.} Bottom element In denotational semantics a partial function is considered as returning the bottom element when it is undefined. In computer science a partial function corresponds to a subroutine that raises an exception or loops forever. The IEEE floating point standard defines a not-a-number value which is returned when a floating point operation is undefined and exceptions are suppressed, e.g. when the square root of a negative number is requested. In a programming language where function parameters are statically typed, a function may be defined as a partial function because the language's type system cannot express the exact domain of the function, so the programmer instead gives it the smallest domain which is expressible as a type and contains the domain of definition of the function. In category theory In category theory, when considering the operation of morphism composition in concrete categories, the composition operation ∘ : hom (C) × hom (C) → hom (C) {\displaystyle \circ \;:\;\hom(C)\times \hom(C)\to \hom(C)} is a function if and only if ob (C) {\displaystyle \operatorname {ob} (C)} has one element. The reason for this is that two morphisms f : X → Y {\displaystyle f:X\to Y} and g : U → V {\displaystyle g:U\to V} can only be composed as g ∘ f {\displaystyle g\circ f} if Y = U , {\displaystyle Y=U,} that is, the codomain of f {\displaystyle f} must equal the domain of g . {\displaystyle g.} The category of sets and partial functions is equivalent to but not isomorphic with the category of pointed sets and point-preserving maps.[2] One textbook notes that "This formal completion of sets and partial maps by adding "improper," "infinite" elements was reinvented many times, in particular, in topology (one-point compactification) and in theoretical computer science."[3] The category of sets and partial bijections is equivalent to its dual.[4] It is the prototypical inverse category.[5] In abstract algebra Partial algebra generalizes the notion of universal algebra to partial operations. An example would be a field, in which the multiplicative inversion is the only proper partial operation (because division by zero is not defined).[6] The set of all partial functions (partial transformations) on a given base set, X , {\displaystyle X,} forms a regular semigroup called the semigroup of all partial transformations (or the partial transformation semigroup on X {\displaystyle X}), typically denoted by P T X . {\displaystyle {\mathcal {PT}}_{X}.} [7][8][9] The set of all partial bijections on X {\displaystyle X} forms the symmetric inverse semigroup.[7][8] Charts and atlases for manifolds and fiber bundles Charts in the atlases which specify the structure of manifolds and fiber bundles are partial functions. In the case of manifolds, the domain is the point set of the manifold. In the case of fiber bundles, the domain is the space of the fiber bundle. In these applications, the most important construction is the transition map, which is the composite of one chart with the inverse of another. The initial classification of manifolds and fiber bundles is largely expressed in terms of constraints on these transition maps. The reason for the use of partial functions instead of functions is to permit general global topologies to be represented by stitching together local patches to describe the global structure. The "patches" are the domains where the charts are defined. See also Analytic continuation – Extension of the domain of an analytic function (mathematics) Multivalued function – Generalization of a function that may produce several outputs for each input Densely defined operator – Function that is defined almost everywhere (mathematics) References ^ a b Christopher Hollings (2014). Mathematics across the Iron Curtain: A History of the Algebraic Theory of Semigroups. American Mathematical Society. p. 251. 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